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CHAPTER IX MEASURING THE LINEAR COORDINATE POINTS OF A PRINT

80. Factors Governing Relative Orientation

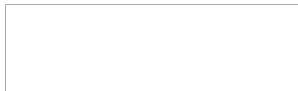
In stereophotogrammetry, allowance has to be made for the effect of the elements of relative orientation on the horizontal parallax difference. To do so, these elements must be understood. As stated previously (see Sect. 74), the elements of relative orientation of a print are identified as the three linear coordinates X_S , Y_S , and Z_S from the center of projection, projections of the angles of tilt α_x and α_y of the optical axis of the camera, relative to the plane coordinates, and of the angle ω denoting the rotation of the print about the optical axis of the camera. A total of 12 elements of exterior orientation must be known for each two prints, i.e., the values

$$X_{S_1}, Y_{S_1}, Z_{S_1}, \alpha_{x_1}, \alpha_{y_1}, \omega_1, X_{S_2}, Y_{S_2}, Z_{S_2}, \alpha_{x_2}, \alpha_{y_2}, \omega_2$$

Obviously, instead of the above-mentioned values for the second print, it is possible to use their difference relative to the first print, or specifically, the values

$$X_{S_2} - X_{S_1}, Y_{S_2} - Y_{S_1}, Z_{S_2} - Z_{S_1}, \alpha_{x_2} - \alpha_{x_1}, \alpha_{y_2} - \alpha_{y_1}, \omega_2 - \omega_1$$

so that, knowing the elements of exterior orientation for the first photograph and the differences described above, it is simple to determine the unknown elements for the second photograph.



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It is obvious (Fig. 14) that the discrepancy of the linear coordinate of the centers of projection for both prints, represents the projection of the distance between the two centers of the projected image frame

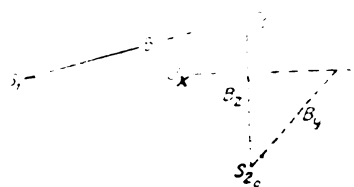


Fig. 14. Projections of the photographic base.

along the axis of the expanded system of coordinates, or in other words, the projections of the photographic base. Thus:

$$X_{S_2} - X_{S_1} = B_x, Y_{S_2} - Y_{S_1} = B_y, Z_{S_2} - Z_{S_1} = B_z \quad (58)$$

applying the fact:

$$B = \sqrt{B_x^2 + B_y^2 + B_z^2} \quad (59)$$

The variances in the values of exterior orientation of the second and first prints determine the position of the prints in space, with respect to each other, and are known as elements of relative orientation. The size of the photographic base does not influence the value of the elements of relative orientation, and determines only the distance between the centers of projection but not their relative location. Thus, five values are applicable to the elements of relative orientation:

$$\frac{X_{S_1} - X_{S_2}}{B} = \frac{X_{S_1} - X_{S_2}}{B}, \frac{Y_{S_1} - Y_{S_2}}{B} = \frac{Y_{S_1} - Y_{S_2}}{B}, \frac{Z_{S_1} - Z_{S_2}}{B} = \frac{Z_{S_1} - Z_{S_2}}{B}$$

as the projection of the base line to the XX axis depends on the values of B and the projection B_y and B_z of the base onto the other coordinate axes.

1. Horizontal condition

Assume that (Fig. 15) the area to be photographed is a horizontal plane that the optical axes of both cameras are vertical, and that the photographic base is horizontal. Then, project through the base line a vertical plane which will intersect



sect the planes of both photographs along a rectilinear xx axis. The direction of this vertical plane will give the XX axis on the area coordinate system. The image

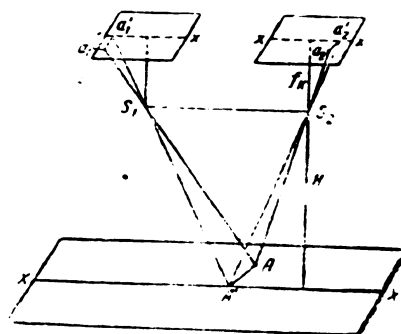


Fig. 150 - Ordinate Values of Corresponding Points at Horizontal Photographic Base

of any point A on the area surface will appear at point a_1 on the left photograph and at point a_2 on the right one. Then drop a perpendicular from points A , a_1 , and a_2 to the corresponding abscissa axis. The foot points of these perpendiculars will be points A' , a_1' , and a_2' . It is obvious that the triangles $S_1a_1a_1'$ and $S_1A'A$ as well as $S_2a_2a_2'$ and $S_2A'A$ are similar in view of the fact that the lines a_1a_1' , a_2a_2' and AA' are parallel. From the similarity of the triangles it follows that:

$$a_1a_1' = v \cdot \frac{A'A}{H} f_k, \quad a_2a_2' = v' \cdot \frac{A'A}{H} f_k$$

i.e., the ordinates of points a_1 and a_2 are equal.

Now, by changing the distance S_1S_2 between the centers of projection, or by shifting both photographs jointly in the direction of the XX , YY , and ZZ coordinate axis, then the equality of the ordinate points a_1 and a_2 will not change. However, the values of the ordinates themselves may change. This corresponding position will remain constant even when both photographs are rotated through equal angles about the base line.

A completely different result is obtained if the position of only one photograph is changed, i.e., if the relative position of the photographs is disturbed. Let us assume that (Fig. 151), the direction of the coordinate axis of the photographs and the area plane have remained the same, but that the right center of projection

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has been shifted upward into the position S'_2 . Then, from the similarity of the triangles $S_1a_1a_1$ and $S_1A'A$ as well as $S_2a_2a_2$ and $S_2A'A$ we can determine the following:

$$y = \frac{A'A}{H} f_k; \quad y' = \frac{A'A}{H'} f_k$$

Since H' is not equal to H , then y' is not equal to y and, therefore,

$$\begin{aligned} y - y' = q &= \frac{A'A}{H} f_k - \frac{A'A}{H'} f_k = + \frac{A'A f_k}{HH'} (H' - H) = \\ &= + \frac{y}{H'} (H' - H) = + \frac{y}{H'} B_z \end{aligned}$$

Similarly, the ordinate points a_2 of y' will change, if the right center of projection is shifted along the YY axis of the area, or if the right print is tilted

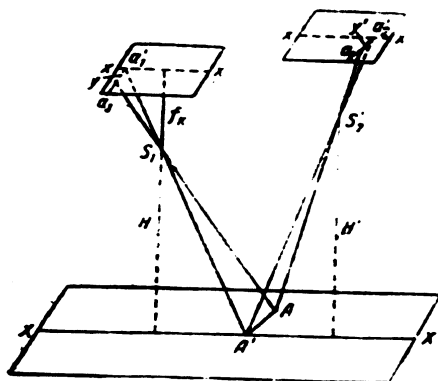


Fig. 151 - Ordinate Values of Corresponding Points at a Tilted Base

through the angles α_{x_2} , ω_2 and rotated through an angle χ_2 . Therefore, the change in position of one photograph relative to the other, i.e., the change in their relative orientation, will disturb the equality of the ordinates of the corresponding points for the two photographs. This discrepancy between the ordinates for the corresponding points of the two prints is known as transverse parallax. The phenomenon of transverse parallax in plan surveying occurs whenever proper relative orientation is not maintained.

Due to the fact that the extent of transverse parallax depends on the elements of relative orientation, it is possible to determine the value of these elements by

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measuring the transverse parallax for a row of points. It was previously mentioned that the horizontal parallax of points had to be measured to determine their difference in height. Consequently, it is necessary to measure both transverse and horizontal parallax, as well as the linear coordinates of points for photogrammetric interpretation of aerial photographs.

82. Stereocomparator

The measurement of linear coordinates of points on a photograph could be accomplished by means of any instrument with a rectangular coordinate system of motion (e.g., a coordinatograph) or even with the aid of a beam compass. However, in view



Fig. 152 - Stereocomparator

of the high degree of accuracy required in measuring transverse and horizontal parallax, the linear coordinates are measured with the aid of precision instruments, using both photographs simultaneously.

The stereocomparator is the basic instrument used for measuring the linear coordinates of points from two photographs. It permits the measurement of the coordinates of one photograph and the difference in the value of the coordinates between

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two photographs. To increase the accuracy of measurements, these are performed on the stereocomparator using the stereoscopic principle of viewing.

The stereocomparator (Fig. 152) consists of a solid base 1, to which the tracks 2 are rigidly attached. These tracks determine the direction of the xx coordinate axis of the instrument. The main carrier 4 is displaced along the tracks by revolving the main shaft 3. The left-hand part of the carrier is provided with a disk 5 composed of a glass plate in a metal frame. On backing off the screw 6, the disk 5 can rotate about its own axis. Minor disk rotations can be made with the aid of a micrometer screw 7. The negative is placed on the glass portion of the disk 5 and is illuminated by a light from below. The right section of the carrier 4 is equipped with cross tracks 8, above which another disk 9 is placed, which can be rotated by hand or by a micrometer screw. The second negative is placed on top the disk 9, which is also illuminated by a light from below.

When the main shaft 3 is rotated, the carrier 4, along with both negatives, moves along the xx axis of the instrument. When the screws 10 and 11, of the cross tracks are rotated, the right negative is shifted either along the xx axis of the instrument (screw 10), or perpendicularly to this axis. These movements accomplish: the displacement of the main carrier along the xx axis, as measured on the scale 12; displacement of one right negative along the xx axis, as measured on the drum-type scale 13; and the displacement of one of the right negatives along the yy axis, as measured on the drum scale 14. The scale 12 is attached to the base and is graduated in minimum units of 0.5 mm. Its vernier scale is mounted to the carrier and has 26 graduations. This permits measuring the movement of the carrier to within 0.02 mm. The drum scales 13 and 14 are divided into 100 graduations each. One complete revolution of the drum moves the right negative by 1 mm. These displacements (in whole millimeters) are indicated on a scale, which is connected to the cross tracks. In this manner, the units of displacement are measured on the scale in whole millimeters, while the drum scales indicate the movement in tenths and hun-

dredths of a millimeter.

Tracks 1.5 are also rigidly attached to the base 1 and serve as the yy axis of the instrument. The binocular microscope 17 is moved along these tracks by means of a hand wheel 16. The eyepieces are located above the negatives. The binocular microscope is composed of two congruent optical systems (Fig. 153), which contain an ocular 1, a floating mark 2, prisms 3 and 4, an objective 5, and a prism 6. The left negative is located beyond the prism 6 (or the right negative, for the right

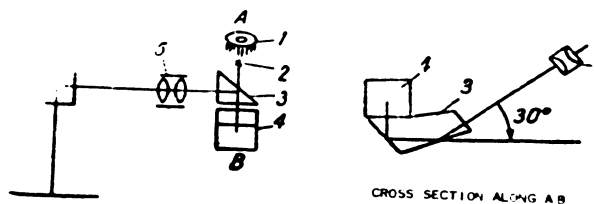


Fig. 153 - Binocular Microscope

section of the binocular), the image of which is formed by the lens 5 in the plane of the floating mark. The floating mark thus appears to coincide with the print. A section of the negative and the floating mark are examined by the observer through the eyepiece 1. Upon sighting the images of the two prints through the binocular, along with the superimposed floating marks, a stereoscopic version of the model of the photographed area is seen. Only one sighting mark is seen and it appears to be floating.

The movement of the suspended floating mark, in one horizontal plane, and in relation to the surface of the stereo image, is accomplished by moving the binocular system along its own tracks in the direction of the yy axis of the instrument. moving the carrier along the xx axis, while the prints and the floating marks remain stationary. With the stereocomparator it is possible to shift one right print along the xx axis to achieve depth distortion displacement of the floating mark. In this way, the stereocomparator permits superposition of the floating mark on any point

common to both prints and thus measurement of their linear coordinates. The distance between the oculars of the binocular microscope is adjustable to compensate for various interpupillary distances of individual observers.

83. Working with the Stereocomparator

The negative pair is placed in the stereocomparator with the emulsion side facing down. In this way, the covered sides of the negatives are both facing the inside of the instrument. Glass plates are placed on top of the negatives and are clamped down with four spring clips. This anchors the negatives in their respective frames.

In positioning, the principal points of the negative are aligned with the centers of rotation of the disks 5 and 9. These centers are marked by crosses on the glass disks. Then, the principal points of the negatives are aligned until the xx axis of the negatives is parallel to the xx axis of the instrument.

The next step is to orient the prints with the stereocomparator. To do this, the carrier is moved along the xx axis, and the binocular microscope along the yy axis, until the reference point on the left negative coincides with the principal point of the left negative. Then, the carrier is moved along the xx axis, and the binocular is held stationary, until the right reference point coincides with the principal point for the right negative. The final adjustment is done by aligning the right reference point with the principal point in the yy axis of the instrument.

Then, a prominent relief feature, located close to the principal point on the left negative is selected, and the left reference point is superimposed with it by moving the carrier along the xx axis and the microbinoculars along the yy axis.

Through this adjustment, the right reference point will automatically coincide with the corresponding relief feature on the right negative. If it does not, an adjustment is made in the horizontal parallax by manipulating its adjusting screw. The same process is repeated for the right negative, with the final adjustment for the

left reference point being made with the transverse parallax screw, and by rotation of the disks to achieve orientation along the vy axis. The xx axis for both negatives should now run in the same direction and parallel to the xx axis of the instrument.

After the negatives have been oriented in this manner, the linear coordinate of the selected relief features can be determined by using the principle of stereovision. In measuring the linear coordinates of the point for both negatives, the observer moves the carrier with the binoculars and matches the reference point with the selected relief point for the left negative. Then, if the right reference point coincides with the selected relief point along the xx axis of the right negative, the floating mark will appear to be higher or lower than the point of the plan. Also if there is misalignment of the right reference point with the relief point, along the yy axis, then the stereovision will be disrupted and both points will appear double. In such a case, the observer must first move the right print along the yy axis until both points appear to be lying in a straight horizontal plane. Next, the floating point is aligned to the point of relief over the horizontal parallax screw. The readings on the scales of the stereocomparator represent the values x , y , P , and Q , which are the linear coordinates for the selected relief point for both prints.

CHAPTER X THE STEREO METER

84. The Function of the Stereometer

When measured on a stereocomparator, horizontal parallax differences in photographs taken from different points, obtained from the angle of tilt of the optical axes and the differences in flight altitudes, depend not only on the relief of the photographed area but also on the magnitude of the elements of exterior orientation. On the stereocomparator, it is not the real value Δp of the horizontal parallax difference but the quantity $\Delta'p$ that is being measured, so that

$$\Delta'p = \Delta p + \delta p$$

where δp is the change in horizontal parallax difference, depending on the magnitude of the elements of exterior orientation. Therefore, to determine the vertical interval of points on the ground by horizontal parallax, the quantity δp must be deducted from the measured values; δp is related to the current coordinates of the points and the elements of exterior orientation by the function:

$$\begin{aligned} \delta p = & \frac{x_{a1}}{f_k} (H - H_0) + \frac{2bx_2}{\rho} \left(\frac{x_{a1}^2}{f_k^2} (x_{x2} - x_{x1}) + \right. \\ & + \frac{x_{a1}y_{a1}}{f_k^2} (\omega_2 - \omega_1) + \frac{y_{a1}}{\rho} (\gamma_2 - \gamma_1 - \frac{b}{f_k} \omega_2) - \frac{2x_{a1}\Delta p}{f_k^2} x_{x2} + \\ & + \frac{2b\Delta p}{f_k^2} x_{x2} - \frac{\Delta p y_{a1}}{f_k^2} \omega_2 + \frac{\Delta p^2}{f_k^2} x_{x2} - \frac{\Delta p}{H} (H_2 - H_0) \end{aligned} \quad (50)$$

The introduction of corrections in measuring horizontal parallax differences may be carried out analytically, if the value of the elements of exterior orientation is predetermined and if the current coordinates of the points are measured. Such a solution, however, is applied only in cases in which the horizontal parallax of a limited number of points has to be corrected. Usually, the automatic introduction of corrections in horizontal parallax differences is used.

An instrument called a stereometer is used for the automatic correction; this is the design of a Soviet scientist, Professor F.V. Drobyshev.

The stereometer consists of a stereocomparator equipped with supplementary correcting attachments which are designed to introduce corrections in the horizontal parallax differences, corresponding to the magnitude of the elements of exterior orientation. There are two types of stereometers for the solution of a given problem: the topographical stereometer and the precision stereometer.

85. Schematic Diagram of a Topographical Stereometer

To the fixed base 1 (Fig. 154) tracks are attached, along which, by turning the rack screw 2, the carrier 3 is moved parallel to the xx axis of the instrument. To the main carrier 3, two other carriers are mounted: the right carrier 4 and the left carrier 5, which also can be moved in the direction of the xx axis of the instrument, independently of the carrier 3. The left carrier 5 is moved along the xx axis by turning the parallactic screw 6, and these changes of position serve for measuring the horizontal parallax of various points of the stereo pair. The right carrier 4 has displacement relative to the elements of exterior orientation and to the distance x traversed by the main carrier 3. These relative displacements are brought about by means of two straight edges 7 and 8 located in a horizontal plane and rotating about a common center. The straight edge 7 is clamped to a caster which is connected with the slide bar 9 (not visible in the diagram); the slide bar may be moved along a track parallel to the yy axis of the instrument and attached to the



main carrier 3. Therefore, when the main carrier 3 is moved, the track with the slide bar and caster is moved, together with the carrier, along the xx axis, forcing

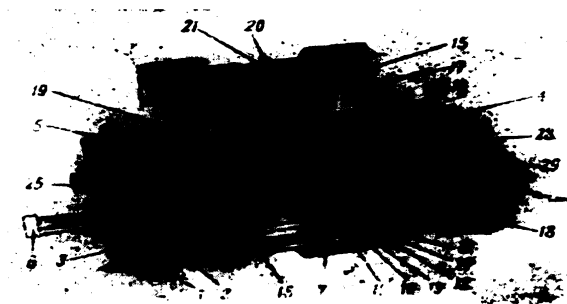


Fig. 154 - Topographical Stereometer

the straightedge 7 to turn about a fixed center through a certain angle i . The angle of rotation i traversed by the straightedge 7 will depend on the distance d

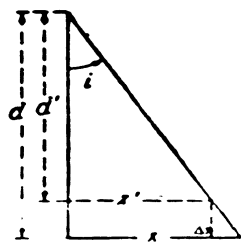


Fig. 155 - Abscissa on Changing the Length of the Straightedge

(Fig. 155) measured along the vv axis of the instrument, between the center of rotation of the straightedges and the center of the caster, and on the segment x traversed by the main carrier. On the basis of Fig. 155, we may write

$$\tan i = \frac{x}{d}$$

The second straightedge 8 is clamped to another caster which can be moved together with the slide bar 10 along the track 11 which is attached to the carrier 4.



Since the second straightedge 8 revolves simultaneously with the first about a common center, it will also rotate through the same angle i . As a result of the rotation of the second straightedge through the same angle i , the right carrier must move along the xx axis by the distance x' related to the angle i by the following equation.

$$x' = d' \tan i$$

where d' is the distance along the yy axis of the instrument from the center of rotation of the straightedges to the center of the caster of the slide bar 10. If, in the initial position, the straightedges 7 and 8 are parallel to each other, a turning of the rack screw 2 will cause both to rotate through the angle i , while the right carrier traverses the segment x' .

Since the movement of the right carrier, together with the main carrier, took place through the segment x , the right carrier must move relative to the main carrier over the distance $x' - x = \Delta x$. From the above equations it is easy to obtain:

$$\Delta x_1 = x' - x = d' \tan i - d \tan i = (d' - d) \tan i = \frac{x(d' - d)}{d} \quad (61)$$

i.e., the displacement of the right carrier relative to the main carrier will be directly proportional to the traversed distance and the difference in length of both straightedges. Therefore, if the length of both straightedges is not the same, the turning the rack screw 2 will cause the left photograph, together with the main carrier, to traverse the segment x , while the right photograph will traverse the segment x' . Scales on the tracks permit reading the distances between the centers of the casters and the center of rotation of the straightedges in a direction perpendicular to the axis xx of the instrument. the scales permit the setting of these distances either equal to each other ($d = d'$) or unequal.

The straightedges may also be set on the instrument at an angle to each other, loosening the clamp screw 12 and moving the straightedge 8 in the slot 13. The magnitude of the angle of rotation of the straightedge 8 can be read off from the scale

14. If the straightedges 7 and 8 are not parallel to each other, then turning the rack screw 2 will cause the carriers to travel different distances x .

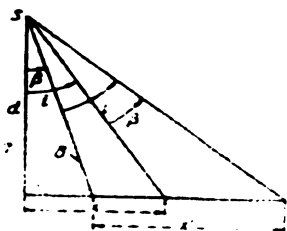


Fig. 156 - Effect of the Angle between the Straightedges

In the initial position (Fig. 156), let the straightedge 7 lie along the yy axis of the instrument and let the straightedge 8 form a certain angle p with it. A displacement of the main carrier along the xx axis by a certain segment x , will cause the straightedge 7 to turn about the center of rotation S through the angle i , determined by the equation:

$$\tan i = \frac{x}{d}$$

The second straightedge turns through the same angle i , which forces the second carrier with the slide bar 10 to move over segment x' , while the angle p between the straightedges remains the same. Since the right photograph, together with the main drum is displaced over the segment x , the independent displacement of the second photograph will be equal to:

$$\Delta x_2 = x' - x$$

From Fig. 156 it follows that:

$$x' = d \tan (i + p) = d \tan p$$

$$\Delta x_2 = d \tan (i + p) - d \tan i = d \tan i = d \frac{\tan i + \tan p}{1 - \tan i \tan p} = d \tan p + d \tan i =$$

$$= d \frac{\tan i + \tan p - \tan p + \tan i \tan^2 p - \tan i + \tan^2 i \tan p}{1 - \tan i \tan p} =$$

$$= d \frac{\tan^2 i \tan p + \tan i \tan^2 p}{1 - \tan i \tan p}$$

(62)

At a small value of the angle i , the second term in the numerator and the sec-

and term in the denominator in the above equation can be dropped, in first approximation, and the angle i can be substituted by its value. Then,

$$\Delta x_2 = d \tan^2 i \tan \beta = \frac{x^2}{d} \tan \beta$$

i.e., the independent displacement of the right photograph alone is directly proportional to the tangent of angle β set between the straightedges, and the square of the traversed distances.

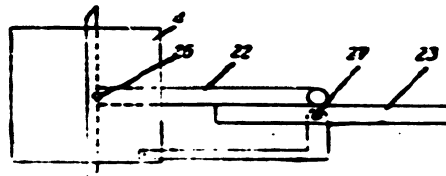


Fig. 157 - Arrangement for Variable Rotation of the Hair

Different points of a stereo pair are observed by means of the viewing device 15 (see Fig. 154), consisting of an ordinary four-mirror stereoscope with lenses. In view of the limited field of view of the stereoscope, it may be displaced along the yy axis of the instrument along the track 16 by turning the rack screw 17. Horizontal parallax is measured with the aid of marks which, in the topographical stereometer, consist of hairs stretched between the hair holders 18. The hairs are stretched over the photograph along the yy axis, which makes it impossible to measure transverse parallax. When moving the carriers 4 and 5 the hairs remain fixed, and the various points of the stereo pair are successively made to coincide with them. To eliminate the transverse parallax which interferes with the stereoscopic effect, one of the lenses of the stereoscope is displaced along the yy axis with the aid of a special screw 19. The equidistance between the exit pupils of the instrument and the eye base of the observer is achieved by changing the distance between

the interior mirrors, which are moved along a support by the slide 20. The corresponding reading is taken on the scale 21.

In addition to the described correctional attachments, based on the difference in length of the straightedges and the included angle β , the topographic stereometer is equipped with other attachments. A third correction device consists of two straightedges 22 and 23 (Fig. 157) located below the carrier 4, with the straightedge 22 attached to the right hair holder which may be rotated about a fixed vertical axis

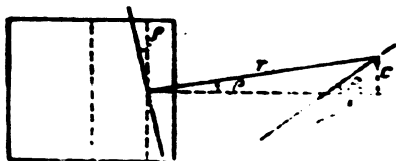


Fig. 158 - Schematic Diagram of the Variable Rotation of the Hair

is 26. The second straightedge 23 may be rotated about a vertical axis 27 attached to carrier 4. Straightedge 22 ends in a caster which is always forced against the straightedge 23 and, when the hair coincides with the principal point on the right photograph it also coincides with the vertical axis 27. If both photographs are moved along the xx axis, the straightedge 23 will also move together with the main

carrier and will slide relative to the fixed caster of the straightedge 22.

Conditions are different when the straightedge 23 is turned in its plane through a certain angle α_0 . Then (Fig. 158), a displacement of the main carrier over the distance x will cause the straightedge 23 to push the caster of the straightedge 22, so that the straightedge 22 and the attached hair holder will turn about the axis of rotation 26 through the angle α . The relation between the angles α_0 and α can be determined from Fig. 158, where the displacement of the caster along the yy axis is indicated by c , and the length of the straightedge 22 (i.e., the distance from the axis of rotation 26 to the center of the caster) is indicated by r . In that case,

$$c = x \tan \alpha_0 = r \sin \alpha$$

and, therefore,

$$\sin \rho = \frac{x}{r} \tan \rho_0$$

Since the abscissa of the points of the right photograph is measured with the aid of the hair, it follows that a rotation of this hair through the angle ρ (Fig. 159) will vary the measured abscissa by the quantity

$$\Delta x_3 = y \tan \rho = \frac{xy}{r} \tan \rho_0 \quad (64)$$

Thus, on displacement of the main carrier along the xx axis of the instrument, the right hair will not remain stationary but will rotate in its plane as a function

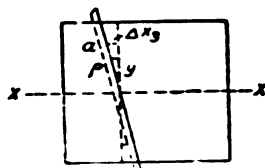


Fig. 159 - Effect of the Angle of Variable Hair Rotation

of the angle of rotation ρ_0 of the straightedge 23 and of the traversed distance. Therefore, the abscissa being measured along the right photograph will vary in direct proportion with the angle of rotation of the straightedge and with the x and y coordinates of the selected point. The given angle of rotation ρ_0 of the straightedge can be read from the scale 28 (Fig. 154).

The fourth corrective attachment of the stereometer is designed for rotating the right hair by a constant angle τ . For this, the vertical axis of rotation 26 of the hair (Fig. 157) has a repeating arrangement which permits the hairs to rotate through the angle τ either together with the straightedge 22 or independently of it. The rotation of the hair, independently of the straightedge 22, is effected by still another straightedge, located underneath the straightedge 22 and rotating, together with the hair, about the vertical axis 26. The lower straightedge, after backing off the set screw, may be turned together with the hair through angle τ , after which it is again rigidly attached to straightedge 22, and participates with it in the rotation through the angle τ .

Therefore, if the angle ρ_0 is equal to zero, the right hair will form the constant angle τ with the yy axis of the instrument, which leads to a change in the abscissa of the right photograph by the following quantity (Fig.160):

$$\Delta x_4 = y \tan \tau \quad (65)$$

For orienting the photographs held by the carriers, it is mandatory that the line joining the principal points of both photographs be parallel to the xx axis of the instrument. Therefore, the carriers with the photographs may be rotated in their planes through the angle χ . The angles of rotation, after the clamp screws (Fig.154) have been tightened, can be read from the scales 25. For convenience in working, an extension shelf 29 is installed in the instrument, on which the observer may rest his arm.

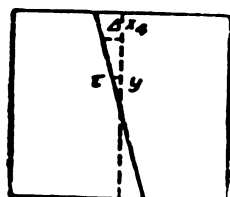


Fig. 160 - Effect of the Angle of Constant Hair Rotation

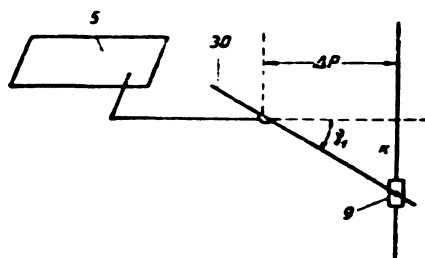


Fig. 161 - First Auxiliary Corrective Attachment

In addition to these four corrective attachments, some topographic stereometers are equipped with two more attachments designed for estimating the combined effect of the angles of tilt and of the ground relief on the change in horizontal parallax difference.

The first auxiliary attachment consists of a straightedge 30, which is held flat against the slide bar 9 and is able to move along the xx axis of the instrument by turning the parallax screw. If, in the initial position, the straightedge

30 forms the angle γ_1 with the direction of the xx axis, then (Fig.161) a shift of the carrier 5 by the distance Δp will cause the slide bar 9 to shift along its track parallel to the yy axis by the distance k , which is determined from the following relation

$$k = \delta d = \Delta p \tan \gamma_1$$

At the same time, the difference δd in the length of the straightedges, affects the independent displacement of the right carriers according to eq.(61), so that

$$\Delta x_5 = \frac{x}{d} \delta d = \frac{x \Delta p}{d} \tan \gamma_1 \quad (66)$$

Thus the independent displacement of the right carrier will be proportional to the abscissa of the observed point, the horizontal parallax difference, and the angle of rotation γ_1 of the correctional straightedge. The value of the angle γ_1 can be read from the scale to within an error of 2 min.

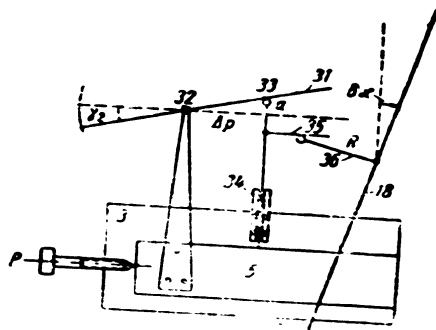


Fig. 162 - Second Auxiliary Correction Attachment

The second auxiliary correction device (Fig.162) consists of a straightedge 31 which may be rotated in the horizontal plane through the angle γ_2 whose axis of rotation is attached to the left carrier 5. This straightedge is always in contact with the castor 33, which is fastened to the straightedge 35 which is displaceable along the track 34, attached to the main carrier 3. The straightedge 35 is brought into contact with the castor by the lever 36, attached to the left carrier 5.

When the main carrier 3 is moved along the xx axis of the instrument, the left carrier 5, the track 34, and the straightedge 35 are also shifted so that the

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straightedge 35, which is always parallel to the xx axis of the instrument, slides along the caster of lever 36. If, however, only the left carrier 5, together with the straightedge 31, is moved, then if the straightedge 31 forms the angle γ_2 with the xx axis, the caster 33 moves along the yy axis by the distance a, determined by the following equation

$$a = \Delta p \tan \gamma_2$$

This shift of the caster 33 leads to a similar displacement of the straightedge 35, which causes the lever 36 and the hair holder 18 to rotate through an angle $\delta\chi$, which is determined by the following expression:

$$\sin \delta\chi = \frac{a}{R} = \frac{\Delta p}{R} \tan \gamma_2$$

where R is the length of lever 36.

When the hair holder is turned through the angle $\delta\chi$, a change in the abscissa of the observed point of the photograph will obey eq. (65) so that

$$\Delta x_6 = y \tan \delta\chi = \frac{y \Delta p}{R} \tan \gamma_2 \quad (67)$$

Thus a change in abscissa in this case will be directly proportional to the ordinate of the observed point, to the parallax difference, and to the adjusted angle γ_2 . The quantity γ_2 can be set up on a scale with scale divisions of $2'$.

86. Orientation of Photographs on the Topographic Stereometer

The topographic stereometer is designed for plotting relief on untransformed photographs and is equipped with correction attachments for automatic correction for horizontal parallax differences. Actually, the correction attachments of the instrument permit measurement of the abscissa on the right photograph:

$$x_2' = x_2 + \Delta x_1 + \Delta x_2 + \Delta x_3 + \Delta x_4 + \Delta x_5 + \Delta x_6$$

as a result of which the measured parallax difference will be equal to

$$\Delta''p = x_1 - x'_2 - b = x_1 - x_2 - b - \Delta x_1 - \Delta x_2 - \Delta x_3 - \Delta x_4 - \Delta x_5 - \Delta x_6 \quad (68)$$

Since the measured parallax difference on the stereocomparator is

$$\Delta'p = x_1 - x_2 - b$$

while the real value Δp of the horizontal parallax difference, is

$$\Delta p = \Delta'p - \delta p$$

then, substituting these values in eq.(68), we obtain:

$$\Delta''p = \Delta'p - \Delta x_1 - \Delta x_2 - \Delta x_3 - \Delta x_4 - \Delta x_5 - \Delta x_6 = \Delta p + \delta p - \Delta x_1 - \Delta x_2 - \Delta x_3 - \Delta x_4 - \Delta x_5 - \Delta x_6 \quad (69)$$

From eq.(69) it follows that, in order to obtain real values for the horizontal parallax difference on the topographic stereometer, the following equation must be satisfied:

$$\delta p = \Delta x_1 + \Delta x_2 + \Delta x_3 + \Delta x_4 + \Delta x_5 + \Delta x_6 \quad (70)$$

or, substituting their values for the above quantities:

$$\begin{aligned} & - \frac{x_{a1}}{f_k} \left(5H + \frac{2b}{d} x_{x2} \right) + \frac{x_{a1}^2}{f_k d} (x_{x2} - x_{x1}) + \frac{x_{a1} y_{a1}}{f_k d} (\omega_2 - \omega_1) + \\ & + \frac{y_{a1}}{f_k} \left(\gamma_2 - \gamma_1 - \frac{b}{f_k} \omega_2 \right) - \frac{2\Delta p x_{a1}}{f_k d} x_{x2} - \frac{\Delta p y_{a1}}{f_k d} \omega_2 + \\ & + \frac{2b\Delta p}{f_k d} x_{x2} + \frac{x_{a1}}{d} (d' - d) + \frac{x_{a1}^2}{d} \tan \rho + \frac{x_{a1} y_{a1}}{r} \tan \rho_0 + \\ & + y_{a1} \tan \tau + \frac{x_{a1} \Delta p}{d} \tan \nu_1 + \frac{y_{a1} \Delta p}{R} \tan \nu_2 \end{aligned} \quad (71)$$

In this expression, the terms $\frac{\Delta p^2}{f_k \rho} \alpha_{x_2}$ and $\frac{\Delta p}{H} (H_2 - H_0)$ are dropped, since the first of these is very small even at a considerable fluctuation in relief, and the second term is equal to zero when $H_2 = H_0$.

It is evident that this equation is valid for any points on stereo pair having different current coordinates, if

$$\left. \begin{aligned} - \frac{\delta H + \frac{2b}{\rho} \alpha_{x_2}}{f_k} &= \frac{d' - d}{d} \\ + \frac{\alpha_{x_2} - \alpha_{x_1}}{f_k \rho} &= \frac{\tan \beta}{d} \\ + \frac{\omega_2 - \omega_1}{f_k \rho} &= \frac{\tan \rho_0}{r} \\ + \chi_2 - \chi_1 - \frac{b}{f_k} \omega_2 \frac{1}{\rho} &= \tan \tau \\ - \frac{2\alpha_{x_2}}{f_k \rho} &= + \frac{\tan \gamma_1}{d} \\ - \frac{\omega_2}{f_k \rho} &= + \frac{\tan \gamma_2}{R} \end{aligned} \right\} \quad (72)$$

The last term $\frac{2b\Delta p}{f_k \rho} \alpha_{x_2}$ is independent of the current coordinates of the observed points and enters as a correction in the measured horizontal parallax differences.

Therefore, if the quantities d , d' , β , ρ_0 , τ , γ_1 , and γ_2 , satisfying eq.(72) are set up on the correction attachments of the stereometer, then the horizontal parallax difference measured on the instrument will be free of errors caused by changes in the elements of exterior orientation. Then, the true horizontal parallax differences will be measured on the topographic stereometer, which differences permit determination of the elevation differences of points on the photographed area and plotting of the relief in the drafting room. Thus, before proceeding to plot-

ting the relief of a photographed area, the correction attachments of the stereometer must first be set in a position corresponding to eq.(72); this setting of the correction attachments is known as orientation of photographs on the stereometer.

Depending on the principle by which the problem is solved, the orientation of the photographs on the stereometer may be performed by two methods: direct and indirect. In the indirect method, the elements of exterior orientation of the photographs are found from special photogrammetric measurements and constructions, and are then used for calculating the calibration values for the correction attachments of the stereometer. In the direct method, the calibration values are found through successive approximations in working on the stereometer itself from known elevations of a few points within the limits of the stereo pair.

The working procedure used in orientation will be somewhat different for plains or mountainous terrain. Consequently, the first problem to be investigated will be that of orienting photographs of level terrain, where only the first four terms of eq.(71) will have an effect on the change in horizontal parallax difference.

In direct photographic orientation on the stereometer, the correct setting of the correction attachments of the instrument is determined by comparing the measured parallaxes with their values obtained from geodetic bench marks.

Assume that the two elevation points A_1 and A_2 were determined geodetically; this makes it possible to obtain their vertical interval from the following equation

$$h = A_2 - A_1$$

Knowing the vertical interval between the points, as well as the elevation and photographic base, the following equation

$$\Delta p = \frac{bh}{H-h}$$

will yield the true horizontal parallax difference. If the horizontal parallax difference Δp between the same points is measured, then the change in parallax differ-

ential due to the effect of the elements of exterior orientation, can be written in the form:

$$\delta p = \Delta'p - \Delta p$$

At the same time, according to eq.(70):

$$\delta p = \frac{x_a}{d} (d' - d) + \frac{x_a^2}{d} \tan \beta + \frac{x_a y_a}{r} \tan \rho_0 + y_a \tan \tau \quad (73)$$

This expression contains four unknown quantities $d' - d$, β , ρ_0 and τ . To find these unknowns, four analogous equations with known changes in horizontal parallax difference must be constructed. To obtain these equations, the elevation of five points on the stereo must be known from which four vertical intervals and, consequently, four changes in horizontal parallax differential can be obtained. This requires the elevations A_1 , A_2 , A_3 , A_4 , and A_5 of five points on the stereo pair, requiring calculation of the four vertical intervals:

$$h_2 = A_2 - A_1; h_3 = A_3 - A_1; h_4 = A_4 - A_1; h_5 = A_5 - A_1$$

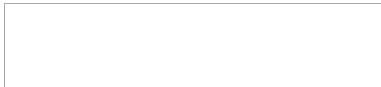
with respect to one of the points taken as the point of origin, as well as measuring of the parallax differences $\Delta'p_2$, $\Delta'p_3$, $\Delta'p_4$, and $\Delta'p_5$ of the same points with respect to the datum point. From the known vertical intervals, the elevation, and the photographic base, the following real horizontal parallax differences are obtained:

$$\Delta p_2 = \frac{bh_2}{H - h_2}; \Delta p_3 = \frac{bh_3}{H - h_3}; \Delta p_4 = \frac{bh_4}{H - h_4}; \Delta p_5 = \frac{bh_5}{H - h_5}$$

and the differences between the measured and calculated values

$$\delta p_2 = \Delta'p_2 - \Delta p_2; \delta p_3 = \Delta'p_3 - \Delta p_3; \delta p_4 = \Delta'p_4 - \Delta p_4; \\ \delta p_5 = \Delta'p_5 - \Delta p_5$$

The resultant changes in horizontal parallax difference permits the construction of



the following system of equations:

$$\left. \begin{aligned} \delta p_2 &= k_1 x_2 + k_2 x_2^2 + k_3 x_2 y_2 + k_4 y_2 \\ \delta p_3 &= k_1 x_3 + k_2 x_3^2 + k_3 x_3 y_3 + k_4 y_3 \\ \delta p_4 &= k_1 x_4 + k_2 x_4^2 + k_3 x_4 y_4 + k_4 y_4 \\ \delta p_5 &= k_1 x_5 + k_2 x_5^2 + k_3 x_5 y_5 + k_4 y_5 \end{aligned} \right\} \quad (74)$$

where

$$k_1 = \frac{d' - d}{d}; \quad k_2 = \frac{\tan \beta}{d}; \quad k_3 = \frac{\tan \rho_0}{r}; \quad k_4 = \tan \tau \quad (75)$$

while the x and y coordinates are read from the principal point of the right photograph. The solution of this system of equations is a rather complicated task; to simplify the procedure, the elevation points must be determined geodetically for specially selected points rather than for random points of the stereo pair. Selection of a special position of points, moreover, makes high accuracy possible. In fact, if denoting the error in the measurement of the horizontal parallax differences by δp and the errors in the coefficients by dk_1 , dk_2 , dk_3 , and dk_4 , then:

$$\delta p = x dk_1 + x^2 dk_2 + xy dk_3 + y dk_4$$

Assuming that $dk_2 = dk_3 = dk_4 = 0$, we obtain

$$dk_1 = \frac{\delta p}{x}$$

Analogously, if $dk_1 = dk_3 = dk_4 = 0$,

$$dk_2 = \frac{\delta p}{x^2}$$

If $dk_1 = dk_2 = dk_4 = 0$

$$dk_3 = \frac{\delta p}{xy}$$

if $dk_1 = dk_2 = dk_3 = 0$

$$dk_4 = \frac{\delta p}{y}$$

It follows from the expressions that the minimum error in the coefficient k_1 will be at x_{\max} ; in the coefficient k_2 , at x_{\max} ; in the coefficient k_3 , at x_{\max} and y_{\max} ; and in the coefficient k_4 , at y_{\max} . At the initial point, the error will equal zero if $x = 0$ and $y = 0$. On the basis of the above considerations, a system (Fig. 163) of a suitable distribution of points having geodetic elevation marks can be constructed. According to this system:

$$x_1 = y_1 = 0; x_2 = \max; y_2 = 0; x_3 = \frac{x_2}{2}; y_3 = 0; x_4 = 0;$$

$$y_4 = \max; x_5 = y_5 = \max$$

Then the equations used for orienting the photographs on the stereometer will take the following form:

$$\delta p_2 = k_1 x_2 + k_1 x_2^2;$$

$$\delta p_3 = k_1 x_3 + k_2 x_3^2;$$

$$\delta p_4 = k_4 y_4$$

$$\delta p_5 = k_1 x_5 + k_2 x_5^2 + k_3 x_5 y_5 + k_4 y_5$$

Since the coefficient k_2 in these equations, according to eqs. (72) and (75), depends on the difference in the longitudinal angles of tilt (α_x) of the two photographs, it follows that usually, even in direct orientation, k_2 is found from other photogrammetric operations, and the angle β is set beforehand on the corrective attachments of the stereometer. In this case, the coefficient k_2 will have no effect on the value of δp , and the basic equation for the orientation will take the form

$$\delta p_2 = k_1 x_2;$$

$$\delta p_4 = k_4 y_4;$$

$$\delta p_5 = k_1 x_5 + k_3 x_5 y_5 + k_4 y_5$$

while the elevation marks of point 3 need not be determined geodetically. Neverthe-

less, successful construction of these equations does not ensure complete accuracy of the operations. Consequently, two additional equations are generally required, compiled from the horizontal parallax points 6 and 7 (Fig. 163) whose elevation must be determined from geodetic measurements. For these points, $x_6 = 0$, $y_6 = -\max$; $x_7 = \max$, $y_7 = -\max$.

Based on the above theoretical considerations, the direct orientation of photographs on the stereometer proceeds as follows: Take two photographs comprising a

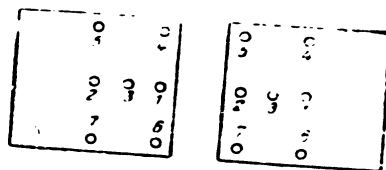


Fig. 163 - Selection of Points for Orientation on Stereometers

stereo pair, within whose borders the elevation marks of six points (1, 2, 4, 5, 6, and 7) have been geodetically determined, distributed according to the above system. For these photos, the following data have been determined beforehand: the mutual longitudinal angle of

of tilt ($\Delta\alpha_x = \alpha_{x_2} - \alpha_{x_1}$); the flight altitude; and the photographic base (the method for finding these values is shown below). Turning the rack screw will shift the main carrier of the stereometer along the xx axis of the instrument until the observer at the stereoscope sees the right hair intersect the centers of rotation of the carriers, marked on their surfaces. In this position, a knot is tied into the hair, which must coincide with the center of rotation. The right photograph is placed on the carrier below the hair in such a way that its principal point coincides with the knot and that the line joining the principal points of both photographs is approximately parallel to the xx axis of the instrument. In the same way, by turning the rack screw, the left hair is made to coincide with the center of rotation of the left carrier. A knot is tied into the left hair, and the left photograph is placed on the carrier. The overlapping parts of both photographs must lie within the instrument and each photograph is then clamped to its respective carrier.

After this preliminary setting, the observer matches the right hair with some contour located near the principal point of the right photograph. If the identical contour on the left photograph does not coincide with the hair in the direction of the xx axis of the instrument, this discrepancy is eliminated by moving the left photograph with the aid of the horizontal parallax screw. If, however, the identical contour seems to be located higher or lower (along the yy axis) than the corresponding contour on the right photograph, the left carrier must be rotated in its plane through the angle χ . In exactly the same way, by matching the left hair with its contour in the vicinity of the principal point of the left photograph, it follows that when the identical point on the right photograph is higher or lower the right photograph must be rotated in its plane through the angle χ . After this, when viewing both photographs simultaneously under the stereoscope, the observer must see a three-dimensional image of the photographed area and only one spatial hair.

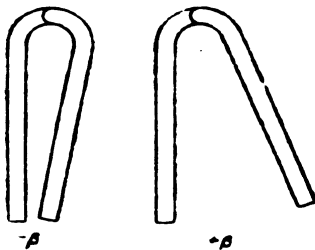
When directly changing to the orientation on the stereometer itself, first the angle β must be adjusted between the correction straightedges and all other correction attachments must be set at zero. For this purpose, the slide bars 9 and 10 are shifted along the tracks to adjust the reading on the d scale to exactly 100 mm, while zero readings are set on the scales τ and ρ_0 . The β is calculated from

$$\beta = (x_2 - x_1) \frac{d}{f_k} \quad (76)$$

If $f_k = 70$ mm, the angle β must be 1.43 times as large as the angle $(x_2 - x_1)$. To set the angle β , the clamp screw must be backed off (Fig. 154) and, while both correction straightedges are held in place manually, the angle between them must be changed to the desired value. When setting the angle β , it should be remembered that it is considered positive when the straightedges are opened and negative when they are closed (see Fig. 164).

After the angle β has been set, the operator, while viewing both photographs in the stereoscope, stereoscopically brings the hair to the observed point so as to

make it appear superposed to the selected point. For more accurate coincidence of the hair with the surface of the model, no elevation control points are pricked on



the working photographs, rather, the observer determines the place at which they are to be located by making pinpricks in the field prints. In measuring horizontal parallax difference the final adjustment is made by turning the parallax screw, which corresponds to a lowering beneath the floating mark.

Fig. 164 - Arrangement of the Angle between the Rulers

The orientation begins from point 1 which is used as the datum point. With

this point, the floating mark is matched twice, and the average of two readings from the horizontal parallax screw is entered in the record of the notebook in which all steps of the orientation are recorded (see Table on p.290). The difference between the two readings should not exceed ± 0.02 mm. If this value is exceeded, the operation must be repeated. In the same manner, the floating mark must be made to coincide with point 2, and the average of two readings entered in the last column of the same chart. The column headed "Should be", is to give the sum of the calculations obtained by superposing the mark on point 1, and calculating from the known interval the horizontal parallax difference of point 2. In the column headed "Difference", the difference between the measured and calculated readings for point 2 is entered. If the obtained difference (δp_2) is more than ± 0.02 mm, it will have to be eliminated by changing the length of one of the corrective straightedges. For this purpose, a reading is taken on the scale of the parallax screw which equals the calculated value entered in the column "Should be". The floating will mark be found not to coincide with point 2 of the model; to correct this, the position of the slide is altered on the track of the corrective straightedge. In order to check the

accuracy of the operations, the floating mark is again superposed with point 2, this time using the parallax screw; if the reading on the scale of the parallax screw is

Table

Data on Photograph Orientation on a Stereometer
Pair No. 1734 - 1735; H = 2750 m; b = 80 mm; $\beta = +1^{\circ}30'$

Point No.	A m	h m	Δp mm	Reading mm	Should be mm	Difference mm	Remarks
1	2	3	4	5	6	7	8
1/1378	458.5	0	0				
2/2411	634.2	+ 175.7	+ 5.46				
4/1 80	385.1	- 73.4	- 2.08				
5/1209	610.4	+ 151.9	+ 4.67				
6/1393	481.3	+ 22.8	+ 0.62				
7/1194	538.2	+ 79.7	+ 2.39				
1				73.48			
2				79.34	78.94	+ 0.40	Δd
1				73.54			
2				79.02	79.00	+ 0.02	
4				71.08	71.46	- 0.38	
1				73.42			
2				78.80	78.88	- 0.08	Δd
1				73.38			
2				78.83	78.84	- 0.01	
4				71.32	71.30	+ 0.02	
5				78.34	78.05	+ 0.29	Δp
1				73.42			
2				78.90	78.88	+ 0.02	
4				71.27	71.34	- 0.07	
1				73.43			
2				78.90	78.89	+ 0.01	$\Delta d = +0.28$ (mm)
4				71.37	71.35	+ 0.02	
5				78.07	78.10	- 0.03	$\Delta p = -0.46$
6				74.02	74.05	- 0.03	
7				75.84	75.82	+ 0.02	

equal to the calculated value (with an error of no more than ± 0.02 mm), then the orientation for this point may be considered complete; if the readings do not agree, the operation must be repeated.

After the calculated readings for point 2 have been established and before proceeding with the orientation, the floating mark must once more be superimposed on the point 1. If the previous reading has changed (which may happen as a result of careless matching of the point 1 with the principal point of the right photograph), a new value for the column "Should be" must be calculated and the above process must be repeated. Thus, the method of successive approximations is used for obtaining congruence of the reading actually measured for point 2 and the calculated value of that reading, preserving the datum reading in the record.

By the same method the floating mark is now matched with point 4, and the average of two readings is entered in the column headed "Reading". Comparison of this reading with the value calculated for it (the sum of the datum reading and the parallax difference calculated from the elevation difference) gives the difference δp_4 , which must be eliminated by turning the hair in its plane through the angle τ .

For this purpose, the calculated value of the reading entered in the column headed "Should be" is set up on the scale of the parallax screw, while the floating mark is matched with the point 4 of the model by rotating the right hair, thus making the rotation equal to the value of the angle τ in eq. (72). To check the correctness of the operations so performed, the floating mark is again matched with point 4, but this time by turning the parallax screw, and if the difference between the measured and calculated readings does not exceed ± 0.02 mm, the orientation with respect to this point is considered completed. If the discrepancy between the readings is over 0.02 mm, the process must be repeated.

After orientation with respect to point 4, the floating mark must again be matched with point 1 and an initial reading taken on the scale of the parallax screw. If there is a significant angle of rotation τ , or if point 1 deviates from the principal point, the initial reading will change, and it will therefore be necessary to repeat the entire orientation process from the beginning, starting with the observation of point 2 and of the change in length of the corrective straight-

edge. By performing the operations in this sequence, the initial reading must be made constant, and the readings directly measured for points 2 and 4 of the model must agree with the values calculated for such readings.

The floating mark is next matched with point 5 of the model, and the average of two readings entered in the column headed "Reading" of the record. If the sum of the initial reading and the calculated parallax difference, as entered in the column headed "Should be", differs from the measured value by more than ± 0.02 mm, a reading equal to the calculated value is set up on the scale of the parallax screw, and the floating mark is matched with point 5 of the model by changing the position of the corrective straightedge ρ_0 . The floating mark is then matched with all four points of the model in the above-described sequence, until the readings on the scale of the parallax screw coincide with the calculated values for all observed points. The floating mark is matched not only with points 1, 2, 4, and 5, but also with points 6 and 7, which constitute the control points. In this case, after orienting the photographs with respect to points 1, 2, 4, and 5, the difference between the measured and calculated readings for points 6 and 7 must not exceed $\pm 0.03 - 0.04$ mm. This order of operations, if the distribution of the points of the stereoscopic pair is sufficiently strict (meaning that the deviation from the standard layout does not exceed 10 mm), permits orientation with only a few approximations. At a large deviation of the position of the points from the scheme, the number of approximation is considerably increased.

It is useful in practical work to first make an approximate orientation, keeping the residual differences to 0.1 mm, and then make the final orientation, using the above-described layout.

All measurements connected with the orientation of a stereoscopic pair, are entered in the record (see p.290), for which the following order of entries has been adopted. In Column 1, the current number of the points is entered, the numerator showing the number of the points in the order of orientation, and the denominator

giving their field numbering.

In Column 2, the altitudes (elevations) of the control points are entered with an accuracy of 0.1 m; in Column 3, their elevations above the datum point; in Column 4, the calculated parallax differences; in Column 5, the readings on the scale of the parallax screw when the space hair coincides with the point in question. The calculated parallax difference, added to the initial reading for point 4, is entered in Column 6 and the difference between the measured and calculated readings, in Column 7.

On completion of the orientation, the observer enters the readings on the scales of the correction devices as well as the values for the photographic base and elevation.

As a result of orientation, the measured horizontal parallaxes of all points of a stereoscopic pair will be free from the effect of the angles of tilt of the photographs and of the difference in flight altitude.

Since, for the orientation of the photographs on the stereometer, it is sufficient to know the elevation mark of four points, the points 4, 5, 6, and 7 often appear as such. In this case, a comparison of measured and calculated horizontal parallax differences of points 4 and 6 permit establishing the value of initial reading and the value of the angle τ , while the elevation marks of points 5 and 7 are used for setting the angle ρ_0 and the difference Δd of the lengths of the straightedges. Such a method of solving the problem slightly changes the order of orientation, since first the angle τ , then ρ_0 , and finally Δd are established.

87. Calibration of the Topographic Stereometer

Before beginning work on the topographic stereometer, all scales of the instrument must be set to zero reading, and the instrument checked and brought into the required condition or, in common terminology, calibrated. This calibration is done partly at the shop, and requires the presence of a mechanic (plant calibration), and

partly during operation of the instrument (service calibration). The plant calibration of a stereometer includes:

1. Matching the centers of rotation of the carriers of both photographs with the intersection of the marker lines on these carriers;
2. Determination of the linearity and parallelism of the track and all carriers;
3. Adjustment of the centers of rotation of both of the two carriers to one line parallel to the x-axis of the instrument;
4. Checking the corrector straightedge 35 (Fig.129) for parallelism with the x-axis of the instrument;
5. Matching the axis of rotation of the carriers to the extension of the axis of rotation of the hair holders, by sighting to the centers of rotation of the carriers;
6. Checking the guides of the corrector or straightedge for perpendicularity to the x-axis of the instrument;
7. Passing the hairs through the centers of rotation of the carriers in the initial position.
8. Checking the image in the stereoscope for quality;
9. Checking the scales and screws for accuracy and true.

To check whether these specifications are met, test grids (Fig.165) are used, consisting of glass plates (13 × 18, 18 × 18, 24 × 24, and 30 × 30 cm) with engraved and traced lines. These horizontal and vertical lines are drawn at intervals of 5 mm, with an error not exceeding 0.01 mm. An indicator consisting of a dial with scale graduations of 0.01 mm and a pin are often used in the calibration. For reading the dial, a pointer connected with the pin over a spring is used. Pressure on the pin causes the pointer to move over the dial. The tracks are checked for linearity and parallelism with the aid of grids, applied to the carrier of the instrument when the central intersection of the grid lines is superimposed on the center of rotation of the carrier. At this instant, a small knot in a hair must coincide with the central

intersection of the grid lines. By turning the rack screw, the main carrier is moved to the left, during which time the hair node must continue to coincide with the same horizontal grid line with which it coincided at the initial position. In case of discrepancy, the lock screw of the carrier is backed off, and, by turning it through the angle χ , the knot is made to coincide with the selected line. The knot must move along the selected horizontal lines, when the rack screw is turned, with an error not over 0.1 mm. Displacement of the knot by more than the tolerance indicates nonlinearity of the track, which must be corrected by the mechanic. Further, if the left carriage is moved with the aid of the parallax screw, while the right carrier is moved by varying the angle β between the corrector straightedges, the knot must be displaced along the same horizontal lines with an error not over 0.2 mm. An error above the tolerance indicates that the three guides lines are out of parallel and must be adjusted by the mechanic.

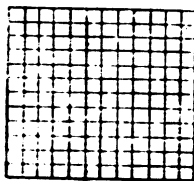


Fig. 165 - The Test Grid

When the carrier with the grid is rotated in its plane through the angle χ , the central intersection of the grid lines must not diverge from the intersection marked on the carrier; any displacement by more than 0.3 mm is corrected by marking a new intersection on the carrier.

The position of the centers of location of both carriers on a single line parallel to the x-axis of the instrument is checked after the two grids have been placed on the carriers so that their horizontal lines coincide with the direction of the x-axis of the instrument. A hair is then stretched over the grids and matched with the horizontal line of one grid. The corresponding line of the other grid must coincide with the stretch hair with an error not over 0.2 mm; any discrepancy is adjusted by the mechanic.

Parallelism of the ruler 35 with the xx axis of the instrument and the setting

of the angle γ_2 are checked by moving the main carrier along the xx axis; in this case, the hair holder must not rotate in its plane.

Perpendicularity of the track of the corrective straightedge to the xx axis of the instrument is checked by moving the slide bar according to special marks located above the left carrier for the orientation grid. These marks must coincide with the vertical line of the grid during additional independent motion of the slide bar, causing a shift of the mark with respect to this line of not more than 0.1 mm for all ranges of motion of the slide bar.

The quality of observation in the stereoscope is checked by stereoscopic observation of grids oriented in the instrument. If corresponding horizontal lines of the grids merge without trouble into a single spatial line, calibration can be considered accomplished. If the corresponding horizontal lines merge or are at an angle to each other, the cause will be a lateral rotation of one of the mirrors.

The position of the axis of rotation of the carriers on the extension of the axes of rotation of the hair holders and coincidence of the hair with the center of rotation is checked after all the correction devices have been set to zero position. Then, after matching the right hair with the center of rotation of the right carrier, the rulers ρ_0 and τ must be rotated to the maximum angles. If on such rotations the hair does not diverge more than 0.2 mm from the center of rotation of the carrier, specifications are considered met; otherwise the mechanic must shift the axis of rotation of the hair holder. The position of the right hair through the center of rotation of the right carriage is checked after setting the x-scale of the instrument to zero position. In this case, the right hair must pass through the center of rotation of the right carrier with an error not exceeding 0.5 mm. If this specification is not met, correction must be made by displacing the center of rotation of the corrector straightedges.

The final check on the scales and operation of the screws of the instrument is made with the aid of an indicator attached to the fixed base of the instrument; the

pin of the indicator must rest on the left carrier. Any displacement of the left carrier by the parallax screw leads to a change in the readings on the indicator scales and the parallax screw; the changes on both scales must be equal.

The above-described operations complete the list of check tests for proper functioning of the instrument and for checking on the accuracy of the plant calibration. Such defects in the instrument can be corrected only by a mechanic.

88. Service Calibration of the Topographic Stereometer

Calibration of a stereometer consists mainly in defining the initial setting and the scale readings of the corrective attachments corresponding to their zero position (i.e., determination of the zero points on the scales of the instrument). Such service calibration of the stereometer is performed periodically with reference to test grids, placed in the instrument's plate holders and oriented along the xx axis. To aid in orientation of the grids, knots are tied in the hairs of the instrument, corresponding with the centers of rotation of the plate holders. Calibration of the topographic stereometer includes the following:

- 1) Determination of the zero point on the x-scale and rotation β ;
- 2) Determination of the zero point on the τ -scale;
- 3) Determination of the zero point on the ρ_0 -scale;
- 4) Determination of the zero points on the d and d' scales;
- 5) Determination of the length r of the radius of rotation of the right hair holder;
- 6) Determination of the zero point on the parallax screw scale;
- 7) Determination of the zero point on the γ_2 -scale;
- 8) Determination of the zero point on the γ_1 -scale;
- 9) Determination of the length R of the radius of rotation of the left hair holder.

Determination of the zero point on the x-scales and the rotation β of one cor-

rective straightedge relative to the other is performed by shifting the tracks of the corrective straightedges in direction of their travel*. To set the zero point on the x-scale, the observer, after shifting the main carrier along the xx axis, matches the right hair with the center of rotation of the right plate holder, and then changes the position of the slide block of the lower ruler. If, during this shift, the right hair moves from the center of rotation of the right plate holder, the position of the main carrier must be changed by shifting it along the xx axis. This sequence of operations is continued until the right carrier remains stationary when the slide block is moved along the lower track, which corresponds to a parallel position of the track and the first corrective ruler. Since, during factory calibration, the track was set along the yy axis of the instrument, the first corrective ruler will occupy the same position. The position of the main carrier thus obtained is set in accordance with the reading on the x-scale and is the latter's zero point.

After the zero point of the x-scale has been determined, the slide block of the upper straightedge is shifted in the same manner, and the motion of the right carrier is eliminated by changing the angle β between the corrective rulers. The scale reading taken for the stationary position of the right carrier corresponds to the mutually parallel position of both corrective rulers and is the zero point of the β -scale. The zero point of the repeating system τ for the hair is read off on its scale after the hair has been matched with a grid line parallel to the yy axis of the instrument, passing through the center of rotation of the right plate holder (Fig. 166).

The zero point on the ω_0 -scale is read off after the hair has been matched with the line on the test grid which is farthest from the center of rotation of the right plate holder (Figure 167). For increased accuracy, the hair is stereoscopically

* In the topographic stereometer ISI-2, the zero point of the x-scale is not determined, but the zero points of the β scale and the differences Δd of the lengths of the rulers are determined by measuring the horizontal parallax differences at three points on the test grid.

matched with the line. To determine the zero points of the d and d' scales of the corrective rulers, their lengths and then their zero points, are adjusted to be equal. First the right hairline is matched with the vertical line on the grid which passes through the center of rotation of the right plate holder, while the left hair

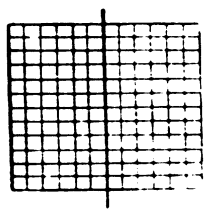


Fig. 166 - Zero Point of the Repeating System

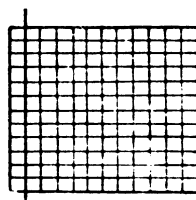


Fig. 167 - Zero Point of Variable Rotation of the Hair

is matched with any line on the left grid. When the lengths of both rulers are equal, the main carrier is shifted along the xx axis until both carriers are equally shifted under the fixed hair (if the zero point was set on the β -scale), i.e., the hairs must coincide with the two other vertical lines of the grid farthest from the original lines. If there is non-coincidence of both hairs with the same vertical lines, this can be remedied by changing the position of one of the slide blocks on its track so that the lengths of both rulers become equal. For determining the zero point of the scales, the right hair is again matched with the vertical line on the grid which passes through the center of rotation of the right plate holder, and then, with the main carriage shifted along the xx axis, it is matched with any other distant line. the corresponding readings of x_0 and x' are taken on the x -scale of the main carrier, while the d and d' readings are taken on the tracks of the corrective rulers. If the reading of d is then changed by sliding one slide block along its track until the hair coincides with another vertical line, i.e., until the right carrier passes through a certain distance δx , then the actual length d_0 can be calculated from the equation:

$$d_o = \frac{(x' - x_o) \Delta d}{\Delta x} \quad (77)$$

where Δd is the variation in reading along the track. Then the zero points of the corrective rulers will represent the difference $(d - d_o, d' - d_o)$ between the lengths of the rulers as read off, and their actual length.

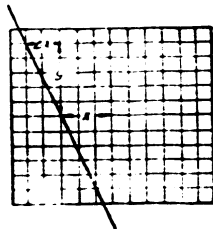


Fig. 168 - Determination of the Length of the Radius of Rotation

The length r , i.e., the radius of rotation of the right hair holder, is determined after the right hair has been matched with any vertical grid line, located at a distance x from the center of rotation of the right plate holder when the reading on the ρ_o -scale is set to correspond with its zero point. If the reading on the ρ_o -scale is changed in such a manner that the end of the right hair coincides with another vertical grid line, then

the length r of the radius can be calculated from

$$r = \frac{xy}{\Delta x} \tan \rho_o \quad (78)$$

where y is the distance along the yy axis to the end of the hair which is matched with the other vertical line (Fig. 168); ρ_o is the established angle of rotation; Δx is the displacement of the end of the hair along the xx axis.

The zero point of the horizontal parallax scale is determined by first matching the hair with the centers of rotation of both plate holders and then reading the corresponding values of x_1 and x_2 on the x -scale of the instrument. The difference $P - (x_2 - x_1)$ between the reading P on the horizontal parallax screw and the value of the displacement $(x_2 - x_1)$ of the main carrier is then the zero point of the horizontal parallax scale.

The zero point of r_1 is determined by rotating the horizontal parallax screw and observing the right hair, which should coincide with some vertical line on the

test grid. If, on such rotation, the right hair does not coincide with the line, the corrective ruler γ_1 is rotated until the difference is eliminated. The reading on the γ_1 scale will then be its zero point.

The zero point of the γ_2 scale is also found by turning the horizontal parallax screw while observing the left hair. If, at first, the left hair coincides with

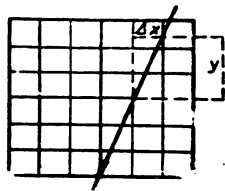


Fig. 160 - Determination of the length of the Auxiliary Corrective Ruler

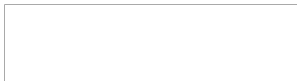
some vertical grid line, and then, after the reading on the horizontal parallax screw has been changed, no longer coincides with the succeeding lines, then the reading on the γ_2 scale must be changed until the hair stops rotating. The reading on the γ_2 scale will then be its zero point. To determine the length R of the radius of rotation of the left hair holder it is necessary to set the maximum reading of γ_2 on the scale and match the hair with some vertical line. After this, turning the horizontal parallax screw will result in a rotation of the hair in its own plane. This rotation is continued until (Fig. 169) the end of the hair, located at a distance y from its center, does not coincide with any but the central vertical line. Then the length R can be calculated from

$$R = \frac{\Delta p v}{\Delta x} \gamma_2$$

where γ_2 is the established angle of rotation, while Δp is the change in reading on the horizontal parallax scale relative to the initial reading.

89. Determination of Camera Station Height and Photographic Base

In orienting the photographs on the stereometer the parallax differences of six points must be calculated from the known geodetic elevation marks and compared



with their measured values. The height of the camera station and the photographic base must be determined in advance before these parallax differences can be calculated.

As demonstrated above (eq.6), for the case of level terrain and vertical direction of the optical axis of the aerial camera, the scale of the aerial photograph is given by the ratio of the focal length of the camera to the flight altitude. Therefore, the flight altitude can be obtained from

$$H = f_k m$$

To determine the scale of the photograph, the ratio of segments drawn between corresponding point-pairs on the photograph and on the ground must be used. Since

$$m = \frac{D}{d}$$

it follows that

$$H = f_k \frac{D}{d} \quad (79)$$

In the case of terrain with prominent relief (but with the photograph still horizontal,) this ratio no longer holds true, because of the fact that the polyconic projection of the photograph differs from an orthographic plan projection, so that a correction for relief must be applied to the distance D. Therefore (Fig.170),

$$D = D_0 + \Delta_1 - \Delta_2$$

where D_0 is the distance between the orthographic projections of the points, and Δ_1 and Δ_2 are the corrections for relief of the two respective ends of the segment selected.

In order that no correction for relief need be applied to the distance D, it is necessary that $\Delta_2 = \Delta_1$, from which it follows that

$$h_2 = \frac{r_1}{r_2} h_1 \quad (80)$$

Thus, if in calculating the flight altitude from eq. (79), the distance between the orthographic projections of the ends of the segment D is taken for the segment D, then the flight altitude will be determined with respect to the plane containing the elevation mark given by the expression

$$A_0 = \frac{r_2 A_2 - r_1 A_1}{r_2 - r_1} \quad (81)$$

where A_2 and A_1 are the elevation marks of the ends of the segment, and r_2 and r_1 their distances from the principal point of the photograph.

This relation is obtained because of the fact that

$$h_2 = A_2 - A_0; \quad h_1 = A_1 - A_0$$

or

$$\begin{aligned} h_2 - A_2 - A_0 &= \frac{r_1}{r_2} h_1 = \frac{r_1}{r_2} (A_1 - A_0) = \\ &= \frac{r_1}{r_2} A_1 - \frac{r_1}{r_2} A_0 \end{aligned}$$

If the optical axis of the camera is tilted, as stated above (Sect. 76), then the segments on the photograph will change with any change in the coordinates of their ends. The variation of these coordinates (without allowing for variation in flight altitude or in angle of rotation of the photograph in its own plane) is expressed by

$$\Delta x = -f_k \tan \alpha_x - \frac{x^2}{f_k} \tan \alpha_x - \frac{xy}{f_k} \sin \omega$$

Therefore, if for the segment on the photograph the distance from a contour point



coinciding with the principal point to any other point is used and if this is adopted as the direction of the xx axis, then the change in the segment d will be expressed by the quantity

$$\Delta x = \frac{x^2}{f_k} \tan \alpha_x \quad (82)$$

which leads to an incorrect determination of the flight altitude. To eliminate the effect of this term on the distance d measured on the photograph, the segments must be selected along a single straight line passing through the principal point of the photograph, while the ends of the segments must be symmetric relative to

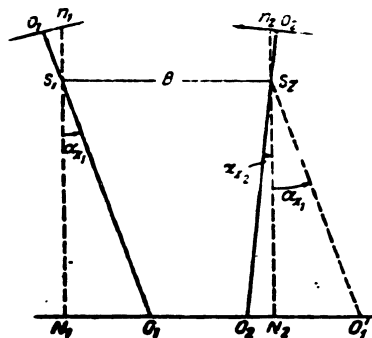


Fig. 171 - Determination of the Photographic Base

this point. Then

$$\left. \begin{aligned} x_1 &= x'_1 - \Delta x_1 = x'_1 - \frac{x'^2_1}{f_k} \tan \alpha_x \\ x_2 &= x'_2 - \Delta x_2 = x'_1 + \frac{x'^2_1}{f_k} \tan \alpha_x \end{aligned} \right\} \quad (83)$$

where x_1 and x_2 are the coordinates of both ends of the segments on the horizontal photograph while x'_1 and x'_2 are those (assumed as equal) of the same points on the tilted photograph. The change in sign in the last term is due to the negative value of x_2 and x'_2 of a point on the other side of the principal point.

Adding the two expressions, and assuming that $x_1 + x_2 = d$, we have

$$d = 2x'_1 = d'$$

i.e., a segment measured on a tilted photograph is equal to a segment measured on a horizontal photograph if the segment passes through the principal point of the picture, and if its ends are equidistant from the principal point.

Therefore, to determine the flight altitude, a segment passing through the principal point, with its ends equidistant from this point, must be selected on the

photograph. This segment as well as the distance between the corresponding points on the plan are then measured, after which the flight altitude is calculated from eq.(79). This flight altitude is obtained relative to a plane of elevation equal to the mean elevation of the segment ends (since $r_1 = r_2$).

The photographic base can be determined from a plan phototriangulation network if the principal point of the photograph is taken for the vertex of direction. In this case (Fig.171), the photographic base is represented by the expression:

$$\begin{aligned} B &= N_1 O_1 + O_1 O_2 - O_2 N_2 = O_1 O_2 + H \tan \alpha_{x_1} - H \tan \alpha_{x_2} = \\ &= O_1 O_2 + \frac{H}{\rho} (\alpha_{x_1} - \alpha_{x_2}) \end{aligned} \quad (84)$$

i.e., the photographic base will be equal to the sum of distances on the map between the principal point and the product of the flight altitude plus the difference in longitudinal angle of tilt. If the plan phototriangulation network has been constructed from working centers not coinciding with the principal points, then such a definition of the photographic base becomes impossible; in that case, the measurements are taken between the principal points of the photographs. Based on eq.(44), the relation between the distances on the photographs and those on the ground is determined by the following equations:

$$\left. \begin{aligned} \frac{f_k}{H} O_1 O_2 &= o_1 o_2' + \frac{o_1 o_2'^2}{f_k} \tan \alpha_{x_1} \\ \frac{f_k}{H} O_1 O_2 &= o_2 o_1' - \frac{o_2 o_1'^2}{f_k} \tan \alpha_{x_2} \end{aligned} \right\} \quad (85)$$

where $o_1 o_2'$ denotes the distance between the principal points on the left photograph, while $o_2 o_1'$ denotes the corresponding distance on the right photograph. Noting that

$$\alpha_{x_2} = \alpha_{x_1} - (\alpha_{x_1} - \alpha_{x_2})$$

and that, since the angles of tilt of the optical axis are low,

$$\tan \alpha_{x_1} = \frac{\alpha_{x_1}}{\rho} \text{ and } \tan \alpha_{x_2} = \frac{\alpha_{x_2}}{\rho}$$

we obtain:

$$\frac{f_k}{H} O_1 O_2 = o_1 o_2' + \frac{o_1 o_2'^2}{f_k \rho} \alpha_{x_1};$$

$$\frac{f_k}{H} O_1 O_2 = o_2 o_1' + \frac{o_2 o_1'^2}{f_k \rho} (\alpha_{x_1} - \alpha_{x_2}) - \frac{o_2 o_1'^2}{f_k \rho} \alpha_{x_1}$$

Adding these two expressions and dividing their sum by two, will yield

$$\frac{f_k}{H} O_1 O_2 = \frac{o_1 o_2' + o_2 o_1'}{2} + \frac{o_2 o_1'^2}{2 f_k \rho} (\alpha_{x_1} - \alpha_{x_2})$$

whence

$$\begin{aligned} b = \frac{f_k}{H} B = O_1 O_2 \frac{f_k}{H} + \frac{f_k}{H \rho} H (\alpha_{x_1} - \alpha_{x_2}) = \\ = \frac{o_1 o_2' + o_2 o_1'}{2} + \frac{o_2 o_1'^2}{2 f_k \rho} (\alpha_{x_1} - \alpha_{x_2}) + \frac{f_k}{\rho} (\alpha_{x_1} - \alpha_{x_2}) = \\ = \frac{o_1 o_2' + o_2 o_1'}{2} + \frac{2 f_k^2 + o_2 o_1'^2}{2 f_k \rho} (\alpha_{x_1} - \alpha_{x_2}) \end{aligned} \quad (86)$$

To determine the photographic base in this manner, the distances between the principal points on the two photographs must be measured, and the mutual longitudinal angle of tilt of the optical axis must be known. The mutual longitudinal angle of tilt is determined by the method given in Section 104.

If the phototriangulation was performed at the vertexes of the directions coinciding with the nadir point, then the measured distance on the network will exactly represent the photographic base. When performing the photogrammetric densification

of the control network by the method of photopolygonometry (see Sect. 112), the values of the height of the camera station are known in advance, and the values for the photographic base, used in calculating the horizontal parallax difference, are expressed by the function

$$b = \frac{Bf_k}{H_1}$$

where B is the value for the photographic base obtained from running the photopolygonometric traverse, while H is the height of the camera station for the left photograph, above the horizontal plane intersecting the projection of the principal point of the right photograph at the ground.

90. Plotting the relief on the Stereometer

Before starting to plot the relief (tracing horizontals) on the oriented stereo pair, a Table of readings from the parallax screw scale corresponding to a given horizontal must be prepared. This table of elevation is computed from

$$P = P_1 + \frac{bh}{H - h} \quad (87)$$

where P denotes the reading on the parallax screw scale for the selected horizontal, which has an elevation difference h above the datum (initial) point which had a reading of P_1 when it was entered.

The first step is to determine the minimum and maximum elevations which are multiples of the sections for the horizontals to be plotted (i.e., 5, 10, or 20 m). To do this, the floating mark is placed on the lowest and highest points of the three-dimensional model (estimated by eye by viewing the entire model), and the elevation differences of these points over the initial point are then calculated from eq. (36). From the known elevation of the first point and from the computed differences in elevation, the elevations of the observed points can be computed within the

limits of a given model. In this way, Column 1 of the Table of elevations can be filled out if the smallest section is taken as a multiple of the assigned and somewhat lower elevation of the lowest point, while the highest value exceeds the elevation of the point with the greatest elevation.

The readings on the parallax screw scale are calculated in the same way for the remaining sections and are entered in Column 2 of the Table. In compiling the Table, computations are made every 50 - 100 m cross section, and intermediate values are obtained by interpolation, as shown in Columns 3 and 4. Differences between sections in the readings of the parallax screw scale obtained by interpolation, are entered in Column 4 of the Table of elevations, which makes it possible to obtain readings for all sections within the selected range.

Table of Elevations
(Stereopair No. 1734-1735)

Height of Horizontal	Computed Value	Interpolation Difference	Difference between Horizontals	Scale Readings
1	2	3	4	5
350	70.39			70.39
360			0.27	70.66
370		1.37	0.27	70.93
380			0.27	71.20
390			0.28	71.48
400	71.76		0.28	71.76
410			0.28	72.04
420			0.28	72.32
430		1.42	0.28	72.60
440			0.29	72.89
450	73.18		0.29	73.18

Before plotting the horizontals, the observer must stereoscopically view the entire area of the stereo pair to interpret properly the general character of the relief. In doing this, special attention should be paid to the hydrographic system

and the direction of divides and thalwegs. In many cases, it is advisable to make such viewing not only on a single stereo pair but for the section as a whole, noting on the photographs the basic elements of relief observed while studying them under a simple stereoscope. Of considerable aid in plotting horizontals is the physico-geographic description of the region, and the standards formulated after studying the shapes and conditions of the formation of relief.

To plot a horizontal, the scale of the parallax screw is set to the value taken from the Table of elevations and from the corresponding horizontals. Then, by moving the main carrier on the xx axis of the instrument, the observer will note that the thread coincides with various points of the stereo model; all these points belong to the selected horizontal. Joining the points on the right contact print with a smooth pencil line gives the horizontal of the given section. After bringing one horizontal to the borders within which relief is being plotted on the stereo pair, a new reading is set on the parallax screw, and another horizontal is then plotted on the contact print. All horizontals on a given stereo pair are drawn in the same manner.

In many cases it is advisable, instead of plotting the horizontals in the manner described above, to measure the horizontal parallax of all characteristic points of the model, and to calculate first the differences in elevation and then their height from eq.(36). The selected elevation marks of characteristic points will make it possible to plot horizontals by interpolation later, using the stereoscopic method of viewing. In this case, no readings are set on the scale of the parallax screw.

The horizontals are drawn according to the technical rules given in official instructions. Besides drawing horizontals, the elevations of points are determined in places specifically indicated in the instructions, e.g., at water boundaries.

Of particular importance in work on the stereometer is checking the accuracy of representation of relief. This is done on completion of the work on the stereo pair

by matching the hair with some point of the model and taking a reading on the scale of the parallax screw. The height of this point calculated from the horizontal parallax should coincide with that read off the plotted horizontal. A second check is made by recording with the adjacent stereo pairs, and also by using elevation control points that had not been used in the stereometer work.

The junctions of the horizontals with the adjacent stereo pairs give their most likely position; this summation of the horizontals can be done in two ways: In the first or visual method, two adjacent photographs already containing the horizontals are placed side by side, and the position of each horizontal compared is checked from the relief. This method should be used for comparatively flat relief, with the horizontals not closer together than 5-10 mm. The second method is the stereoscopic method of plotting the horizontals, which is done under a simple stereoscope. After an analysis of the recording, the new position of the horizontal is marked and should correspond to the marked relief.

91. Processing of Prints of a Mountainous Terrain

The processing of photographs of mountainous terrain on a stereometer has specific features due to the fact that the instrument is equipped with auxiliary corrective attachments. The main corrective attachments of the stereometer, as shown in Section 86, take into account the influence of the first four terms of the above equation, while the last two terms refer to the auxiliary attachments. However, when the relief of the terrain is pronounced, these terms may strongly influence the variation in horizontal parallax. For example, for photography at an altitude of 3500 m with a base on the survey scale equal to 70 mm, at an elevation difference of the selected point over the datum point of 750 m, $\gamma_{x_2} = 2^\circ$, $f_k = 70$ mm, and $x_1 = 0$, the horizontal parallax difference will be 15 mm, while the influence of the next to the last term will be 1 mm, corresponding to an elevation difference of 50 m. Thus, at pronounced relief, the failure to allow for the last two terms in eq. (72) will

lead to errors in elevation amounting to several tens of meters.

Therefore, for cases of considerable fluctuation in the relief of a photographed area, the orientation of the photographs on the stereometer and the plotting of the relief must be performed with the angles γ_1 and γ_2 set on the correction attachments so as to satisfy eq.(72). At the same time, it is evident that the values of the angles γ_1 and γ_2 depend on the elements of exterior orientation, the angles α_{x_2} and ω_2 , with which the corrective terms Δd and τ are correlated. Therefore, assuming in first approximation that $\Delta H = 0$ and $\alpha_2 - \alpha_1 = 0$, we can write:

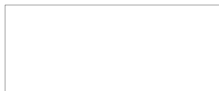
$$\left. \begin{aligned} \alpha_{x_2} &= - \frac{\Delta d f_k \rho}{2db} \\ \omega_2 &= - \frac{f_k}{b} \tau \end{aligned} \right\} \quad (88)$$

The conclusion as to equality of the angles α_1 and α_2 and equality of the flight altitudes can be reached since, on orientating the photographs on the stereometer according to their initial direction, the difference between the angles α_1 and α_2 will be very small, while the difference in flight altitudes of two adjacent photographs of the same flight strip will also always be small. Of course, such an assumption will lead to some errors in the determination of the angles α_{x_2} and ω_2 , but these are minor.

On the basis of eq.(88) for the angles α_{x_2} and ω_2 and of eq.(72), we can write:

$$\begin{aligned} \tan \gamma_1 &= - \frac{2d}{f_k} \alpha_{x_2} + \frac{2d\Delta d f_k \rho}{f_k \rho d 2b} + \frac{\Delta d}{b} \\ \tan \gamma_2 &= - \frac{R}{f_k \rho} \omega_2 = \frac{R \tau f_k}{f_k \rho b} = \frac{R}{b \rho} \tau \end{aligned}$$

Therefore, the processing of photographs of mountainous terrain on the stereometer must be performed in the following sequence:



After placing the photographs on the stereometer, they are oriented by the above method with an accuracy to within 0.1 mm, but uneliminated residual errors for the control points may still remain. After this, readings of Δd and ω are taken from the scales of the instrument, making it possible to calculate the angles γ_1 and γ_2 and the corrective term $\delta''p$ from the equations:

$$\left. \begin{aligned} \gamma_1 &= \frac{\Delta d}{b} \rho \\ \gamma_2 &= \frac{R}{b} \tau \\ \delta''p &= + \frac{2b\Delta p}{f_k \rho} a_{x_2} = - \frac{2b\Delta p \Delta d f_k \rho}{f_k \rho d 2b} = \\ &= - \frac{\Delta d}{d} \Delta p \end{aligned} \right\} \quad (89)$$

while the last corrective term is calculated from the calculated horizontal parallax difference for all control points.

The calculated values of the angles γ_1 and γ_2 are set on the correction attachments of the instrument, while the calculated horizontal parallax differences of all control points are corrected by the value of $\delta''p$. This setting makes it possible to perform a secondary orientation of the photographs on the stereometer according to the above system, as a result of which there should be no residual errors at the control points. The form for an orientation record is shown in the following Table. Columns 1, 2, 3, and 4 are filled out before the first orientation and Columns 5 and 6, before the second. Then the relief is plotted under the stereometer by a method analogous to that described above, except for the fact that, in calculating the readings of the horizontal parallax screw, the correction $\delta''p$ must be taken into account. Therefore, in calculating the Table of elevations (given below), the horizontals are entered in Column 1, the elevation differences of a few contours relative to the datum point are entered in Column 2, the calculated values of hori-

Record of Photograph Orientation on the Stereometer

Pair No. 1841 - 1842

 $H = 2800 \text{ m}; b = 70 \text{ mm}; \beta = 2^\circ$ $f_k = 70 \text{ mm}$

Point No.	A m	h m	Δp mm	$\delta'' p$ mm	$\Delta p + \delta'' p$ mm	Reading mm	Should be mm	Difference mm	Remarks
1	2	3	4	5	6	7	8	9	10
1/58	637.2	0.0	0.00		0.00				$d = 100 \text{ mm}$
2/17	841.7	+ 204.5	+ 5.50	- 0.07	+ 5.43				
4/52	254.3	- 382.9	- 8.42	+ 0.10	- 8.32				
5/24	468.7	- 168.5	- 3.97	+ 0.05	- 3.92				
6/63	544.1	- 73.1	- 2.25	+ 0.03	- 2.22				
7/35	896.2	+ 259.0	+ 7.13	- 0.09	+ 7.04				
1						79.15			
2						83.83	84.65	- 0.82	Δd
1						79.21			
2						84.68	84.71	- 0.03	
4						70.93	70.79	+ 0.14	τ
1						79.19			
2						84.68	84.69	- 0.01	
4						70.75	70.77	- 0.02	
5						75.47	75.22	+ 0.25	ρ_o
1						79.22	-	-	
2						84.71	84.72	- 0.01	
4						70.82	70.80	+ 0.02	
5						75.26	75.25	+ 0.01	$\rho_o = -0^\circ 24'$ $\Delta d = + 1.24$
6						76.79	76.97	- 0.18	$\tau = -2^\circ 17'$
7						86.10	86.35	- 0.25	$\gamma_1 = +1^\circ 01'$ $\gamma_2 = -2^\circ 36'$
1						79.20			
2						84.51	84.63	- 0.12	Δd
4						70.96	70.88	+ 0.08	τ
1						79.21			
2						84.63	84.64	- 0.01	
4						70.91	70.89	+ 0.02	
5						75.20	75.29	- 0.09	ρ_o
1						79.20			
2						84.65	84.63	+ 0.02	
4						70.91	70.88	+ 0.03	$\rho_o = -0^\circ 17'$
5						75.26	75.28	- 0.02	$\Delta d = + 1.34$
6						76.97	76.98	- 0.01	$\tau = -2^\circ 03'$
7						86.27	86.24	+ 0.03	

horizontal parallax differences in Column 3, the correction for calculated parallax difference in Column 4, and the sum of the calculated difference plus the correction in Column 5. The difference between the obtained values is taken, entered in Column 6, and interpolated among the remaining sections. The result of this interpolation is entered in Column 7, which makes it possible to obtain readings on the horizontal parallax screw, adding the obtained values to the initial reading.

92. The Precision Stereometer

In a number of cases, for work requiring high accuracy in measuring the horizontal parallax difference, a precision stereometer is used. A precision stereometer is the same general design as a topographic stereometer. The basic difference



Fig. 172 - The Precision Stereometer

between a precision stereometer and a topographic stereometer are: use of points as measuring marks instead of cross hairs; use of a binocular microscope like that in a stereocomparator; use of negatives instead of contact prints for measuring; and a special system for the corrective attachment ρ_0 .

To the base 1 (Fig. 172) of the precision stereometer, tracks serving as the xx axis of the instrument, are mounted. The main carrier 3, moved by turning the hand-

wheel 2, is displaced along these tracks. A second pair of tracks parallel to the first, is located to the right of the carrier 3. The upper carrier 4, which bears

Table of Elevations
Pair No. 1841 - 1842
b = 70 mm; H = 2800 m;
d = 100 mm; $\Delta d = 1.34$ mm

Contour Reading m	Difference m	Interpolation					Scale Readings m
		Calculated Δp mm	Correc- tion δp mm	$\Delta p + \delta'' p$ mm	Difference ($\Delta p + \delta'' p$) for excess of 100 m mm	Difference between Horizontal mm	
1	2	3	4	5	6	7	8
250	- 387	- 8.50	+ 0.11	- 8.39	+ 1.97	0.39	70.81
270						0.39	71.20
290						0.39	71.59
310						0.39	71.98
330						0.40	72.38
350	- 287	- 6.50	+ 0.08	- 6.42	+ 2.09	0.40	72.78
370						0.41	73.19
390						0.41	73.60
410						0.42	74.02
430						0.42	74.44
450	- 187	- 4.38	+ 0.05	- 4.33	+ 2.25	0.43	74.87
470						0.44	75.31
490						0.45	75.76
510						0.45	76.21
530						0.46	76.67
550	- 87	- 2.10	+ 0.03	- 2.07	+ 2.39	0.46	77.13
570						0.47	77.60
590						0.47	78.07
610						0.48	78.55
630						0.48	79.03
650	+ 13	+ 0.32	0.00	+ 0.32	+ 2.62	0.49	79.52
670						0.50	80.02
690						0.51	80.53
710						0.52	81.05
730						0.54	81.59
750	+ 113	+ 2.98	- 0.04	+ 2.94		0.55	82.14

the negative holder 5 with the attached negatives, can be moved along these secondary tracks. The carrier 4 is shifted along the xx axis of the instrument by means of the parallax screw 6, with the resultant displacement read from the scale and drum 7, with an accuracy up to 0.01 mm. Besides moving along the xx axis, the right carrier can also move in direction of the yy axis of the instrument, independently of the left carrier. This movement is accomplished by means of the transverse parallax screw 8.



Fig. 173 - Corrective Attachment of the Precision Stereometer

The left negative is placed into the plate holder 9, which is placed on the left carrier 10, which also can be moved independently of the right carrier along tracks located on the main carrier 3. This displacement is effected by the corrective attachments of the instrument, and is intended to compensate the influence of the elements of the exterior orientation of both photographs on the change in horizontal parallax. The correction attachment, used for compensating the effect of the difference in flight altitude and the longitudinal angles of tilt of the two

photographs, consists of two vertical straightedges, rotating about a fixed axis, on which the graduated circle 11 for calculating the angles of rotation of the straightedges (Fig. 173) is located. The straightedge 12, over a scissors arrangement is pressed against a roller, located on the moving part of the support 13. Support 13 is rigidly attached to the main carrier 3 of the stereometer. The second straightedge 14, again over a scissors arrangement, is pressed to a roller, connected with the movable part of the support 15. This support 15 is connected with the left carrier 10. The position of the rollers of the support can be changed separately relative to the carriers by turning the small drums 16 and 17, which makes it possible to change the length d of the corrective rulers. By turning the screw 18, the straightedge 14 can be rotated through the angle β ; correspondingly, the reading is taken from the scale of the graduated circle 11.

The precision stereometer is equipped with a special correction attachment to allow for the influence of the lateral angles of tilt, which rotates the left photograph in its plane through the angle ρ . A vertical shaft is mounted to the stand 1 about which the ruler 19 is rotated in its plane through the angle ρ_0 . After the ruler 19 (Figs. 172 and 174) is turned, it is fixed in its new position, and the rotation is read off the scale. A roller, ending in the bar 20 and placed in a horizontal plane, is pressed against ruler 19 by scissors. The other end of the bar 20 is connected to the bracket 21, which is fastened to plate holder 9 which can be rotated in its plane about a spindle passing through its center. When the ruler 19 is turned and the main carrier 3 is moved along the xx axis of the instrument, the roller of the bar 20 will move on the yy axis of the instrument, which will cause the plate holder 9 to turn through the angle ρ . The final corrective motion is effected by rotating the plate holder in its plane through the constant angle τ .

The observation of various points of the photographs on a precision stereometer is carried out with the aid of marks, located in the focal planes of the eyepieces of the binocular microscope. The moving part 22 of the binocular microscope is

1

moved by the handwheel 23 on the yy axis of the instrument. The fixed part 24 of the binocular microscope is rigidly attached to the base 1 of the stereometer. The observation system of the stereometer has a magnification of about 3.5. The work on the precision stereometer is exactly the same as that described above for the topographic stereometer. The instrument is often used as a stereocomparator, with the correction attachment set at the initial (zero) position.

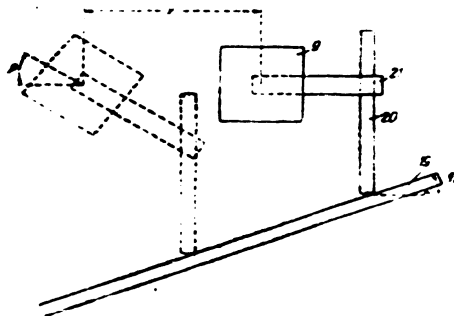


Fig. 174 - Schematic Diagram of the Variation in Rotation of the Photograph

Since the precision stereometer contains no auxiliary correction attachments for the combined influence of relief and angles of tilt, another method must be used for processing photographs of mountainous terrain. In this case, after the first orientation, the equation

$$\epsilon' p = \frac{2(x-b)\Delta p}{f_k^2} \cdot x_2 = \frac{v\Delta p}{f_k^2} \cdot 2 = \frac{(x-b)\Delta p \cdot d}{bd} = \frac{v\Delta p}{b_s} \quad (90)$$

is used for calculating the corrective terms in the horizontal parallax difference of all control points; this makes it possible to carry out a final orientation. In plotting the relief, instead of automatic application of the corrections δd and δt ,

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Record of Orientation of Photographs on a Stereometer

Pair No. 1844-1845 H = 2800 m; b = 70 mm; $\beta = +1^\circ$

Point No.	A m	h m	Δp mm	$x_2 - x_1 - b$ mm	y mm	$\delta^1 p$ mm	$\Delta p + \delta^1 p$ mm	Calculated mm	Should be mm	Difference mm	Remarks
1	2	3	4	5	6	7	8	9	10	11	12
1/34	1254.2	0.0	—	—	—	—	—	—	—	—	—
2/26	383.8	-870.4	-16.60	+ 7	- 4	- 0.06	-16.66	—	—	—	—
4/48	875.5	-378.7	- 8.34	+74	+65	- 0.09	- 8.43	—	—	—	—
5/136	1185.4	- 68.8	- 1.68	- 3	+70	+ 0.07	- 1.61	—	—	—	—
6/88	1319.4	+ 65.2	+ 1.67	+68	-72	+ 0.09	+ 1.76	—	—	—	—
7/41	1241.6	- 12.6	- 0.35	+ 8	-81	- 0.02	- 0.37	—	—	—	—
1								79.34	—	—	—
2								62.96	62.74	+ 0.22	Δd
1								79.30	—	—	—
2								62.73	62.70	+ 0.03	—
4								70.88	70.96	- 0.08	—
1								79.28	—	—	—
2								62.70	62.68	+ 0.02	—
4								70.91	70.94	- 0.03	—
5								77.45	77.60	- 0.15	ρ_o
1								79.31	—	—	—
2								62.77	62.71	+ 0.06	Δd
1								79.30	—	—	—

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STAT

Continuation

Point No.	A m	h m	Δp mm	$x_2 - x_1 - b$ mm	y mm	$\delta' p$ mm	$\Delta p + \delta' p$ mm	Calculated mm	Should be mm	Difference mm	Remarks
1	2	3	4	5	6	7	8	9	10	11	12
2								62.71	62.70	+ 0.01	
4								70.94	70.96	- 0.02	$\Delta d = + 1.45$
5								77.64	77.62	+ 0.02	$\mu_o = -0^{\circ}55'$
6								83.34	80.97	+ 0.37	$\tau = 2^{\circ}30'$
7								79.38	78.95	+ 0.43	
1								79.27	-	-	
2								62.52	62.61	- 0.09	Δd
1								79.28	-	-	
2								62.60	62.62	- 0.02	
4								70.94	70.85	+ 0.09	τ
1								79.25	-	-	
2								62.61	62.59	+ 0.02	
4								70.85	70.82	+ 0.03	
5								77.70	72.64	+ 0.06	μ_o
1								79.22	-	-	
2								62.55	62.56	- 0.01	
4								70.82	70.79	+ 0.03	$\Delta d = + 1.22$
5								77.59	77.61	- 0.02	
6								80.94	80.98	- 0.04	$\mu_o = -0.37$
7								78.81	78.85	- 0.04	$\tau = -2^{\circ}40'$

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Table of Elevations and Setting of the Correction Attachments

Pair No. 1844-1845; $b = 70$ mm; $H = 2800$ mm; $d_o = 101.22$; $i_o = 357^{\circ}20'$

Reading mm	Excess mm	Δp mm	δd mm	δt	$\delta'' p$ mm	$\Delta p + \delta'' p$ mm	Interpolation Difference mm	Difference between Horizontals mm	Scale Reading mm
1000	-254	-5.82	-0.10	+ 13'	+ 0.07	- 5.75		0.41	73.47
1020								0.42	73.88
1040							+ 2.14	0.43	74.30
1060								0.44	74.73
1080								0.44	75.17
1100	-154	-3.65	-0.06	+ 8'	+ 0.04	- 3.61		0.45	75.61
1120								0.46	76.06
1140							+ 2.30	0.46	76.52
1160								0.46	76.98
1180								0.46	77.44
1200	- 54	-1.32	-0.02	+ 3'	+ 0.01	- 1.31		0.47	77.91

they are applied manually on the basis of calculations according to

$$\left. \begin{aligned} \delta d &= \frac{\Delta d \Delta p}{b} \\ \delta \tau &= \frac{\tau \Delta p}{b} \end{aligned} \right\} \quad (91)$$

which follows from the fact that during operation of the correction attachments for Δd and τ , the change in the abscissa takes place according to the equation

$$\Delta x = \frac{x}{d} \delta d + y \frac{\delta \tau}{\rho} \quad (92)$$

A comparison of eqs.(90) and (92) readily indicates the accuracy of eq.(91).

On the basis of the above schematic diagram, a standard form for the orientation record is shown in the Tables given below.

In this record, Columns 1, 2, 3, and 4 are filled out after the first orientation, after which corrections $\delta'p$ are determined from eq.(90). This permits a second orientation. A Table of elevations, in the form shown below, is also prepared.

Thus, the changes δd and $\delta \tau$ are calculated only for certain sections, in order that their differences do not exceed 5 - 6'; and the values are set on the scale of the corrective attachments only for these sections. Intermediate sections are obtained with the same settings, without perceptible errors in the final result. The quantity $\delta''p$ is calculated by interpolation for all sections.

The precision stereometer is used basically for measuring horizontal and transverse parallax and for measuring the angles between the initial directions.

CHAPTER XI

HOW TO MAKE THE ORIGINAL OF A MAP

The end product of the related processes of aerial photography, geodetic surveys, topographic surveys, photogrammetry, and cartographic work is a map, on which are represented, in their proper perspective, all the vital characteristics of the area.

There are several different methods of compiling an original map. The selection of any one method is governed by the technical and economic conditions at hand and by the characteristics of the topography of the mapped area.

93. Preparing a Map of Flat Terrain by Making a Photomap

A photomap or mosaic is usually the basis for making a map of flat terrain. The terrain interpretation and surveying (or the stereoscopic plotting) of the relief can be performed at this stage or directly on the photomap, or on the contact print, and sometimes even on the mosaic itself.

The photomap mosaic, obtained by assembly of aerial photographs, is usually preserved as the original. The photomap is reproduced in many copies for further field work, for field surveying of the terrain and also for ultimate exploitation by various users. Reproduction is done by photographing the mosaic, while strictly adhering to accurate specifications as to size of the negative frame; then, the mosaic is printed on photographic paper, in the required number of copies. For accurate work, the photographic paper is first glued to a rigid aluminum base.

If the terrain interpretation and relief surveying are done directly on the photomap copy, the work copy of the photomap (reproduction) with the drawn-in contour lines becomes the original of the map. From this original a blueprint is prepared which is used for drafting the copy for printing purposes. It is possible to bleach out the photographic image reproduced on the photomap with drawn-in contours. This is done by immersing the photomap into a 5% potassium ferricyanide solution. After the photographic image has disappeared from the photomap, the mounted map is washed and placed into a solution of hyposulfite for 5 min, and then washed again.

To prevent the India ink from washing off during the above-mentioned bleaching process of the photomap, about six to eight drops of formalin are added to each small bottle of India ink.

Terrain interpretation and surveying (or the stereoplotting) of the relief on the contact prints, using the photomap (for flat terrain), will raise the problem of transferring the layout and relief from the prints to the photomap. This problem can be solved in several ways.

In areas of bold relief, where it is easy to locate and transfer the different points from a print onto a photomap, the elements of the layout and the horizontals can be defined by projecting the contours of the print and the photomap. However, if the terrain lacks bold contrast on the prints, the relief and layout can be determined with stereoscopes which permits the viewing of two images from prints of different scales, as is the case in the stereoscope developed by engineer Bashtan*.

The steps of this process will be described below. After measuring equal ground distances, as represented on the print and on the photomap, the comparable magnification factors are next determined and corresponding lenses are installed in the stereoscope. These are moved over the photomap until the image of the contours, located near the principal point on the print, coincides with the corres-

*See description of the Bashtan stereoscope, Sect. 70.

ponding contours on the photomap, and the contour points on the rest of the print are properly oriented in relation to their corresponding points on the photomap. Next, the print is moved along the XX axis of the instrument, using the large mirror. The lens, through which the photomap is viewed, is moved along the ZZ axis. The contour images of the print and the photomap are thus superimposed. After this, the detailed layout and the contour lines of the print, visible as superimposed contour lines on the photomap, are traced on the photomap itself.

It is of interest that, in using the Bashan stereoscope, the transformed image of the photomap is superimposed with the image of the print, which is distorted due to the tilt at the time of photographing. Consequently, the contours of the print can be superposed onto the photomap only when this angle of tilt is negligible. Under adverse conditions, however, the contours are superimposed a section at a time. The contours and other details in such cases are also plotted in by sections.

During this sectional tracing it is advisable to keep checking on proper coincidence of the contour lines between the sections.

After plotting, the photomap is retouched and corrected on the basis of field surveys, to confirm completeness and correctness of the contours transferred to the prints.

Next a blueprint of the photomap is made which is then submitted for reproduction processing.

To transfer contour lines and terrain interpretations from prints onto photomaps, other instruments can be used such as pantograph, projector, or plotter. Use of these instruments is described below.

94. Preparing the Graphic Map

In making maps of flat terrain, as indicated above, the usual method is to prepare first a photomap. However, even in the case of flat terrain it is absolutely necessary to prepare the map from aerial photographs, especially in renovations

of obsolete maps or in using different aerial photographs (as in warfare).

In this case, all elements of the relief plan are transferred to the basic plan, where they are drawn in from field data or from photointerpretation, or else from field or photomap surveys of the relief.

To transpose the contours and horizontals from the photoprint to the base, i.e., to orient the print with the base and to interpolate its form to the scale of the map, a network matching control points is selected on the photograph and on the map. At least four control points are necessary for calculating the transformation. These control points can be located with the aid of phototriangulation or by photographic traverse surveying. For revision of obsolete maps, the control points can be substituted by unchanged contour points on the map, which can be identified on the photographs.

The actual transfer of the horizontals and contours from the photograph to the map is accomplished by a projector or a pantograph. The most commonly used instrument for transferring to maps is the projector which is able to transfer the image of the photograph to the map and eliminates the common mistake of confusing the relief outlines with the contour lines. A pantograph does not transform the photograph, so that distortion occurs in the elements which transfer the relief outlines and contours, caused the tilt distortion of the photograph. Therefore, when using a pantograph, the photographs are handled in sections of limited size so that the marginal error will not be large with respect to the map. In this case, the tracing of the sections is done after the photograph is reduced, with the limits of the control points thus representing the true area of the section on the photograph.

Occasionally, on field trips the only available instrument is the pantograph. In using the pantograph the scale ratio and orientation of the photograph is achieved with two of the control points, the results being checked with the rest of the control points (of the entire print or a section of it). After superimposing the control points, the tracing of contours and horizontals is done. The pantograph

does not permit simultaneous observation of all the control points (and contours) of the print and map, which lowers the accuracy of establishing the proper scale and orientation of the print, and thus lowers the final quality of the map.

*95. Developing the Graphic Plan of Areas with Bold Relief

For areas with bold relief, for cases in which superposition of control points of the relief is used, the number of sections resulting from the large number of such control points, makes sectional transformation an impractical process so that no photomaps are made. In such cases, a graphic plan (map composite) is prepared from aerial photographs, on which contours and horizontals are plotted.

A composite graphic plan from photographs can be prepared by using different prints (projector) or by constructing a topographic model of the area with the aid of a stereo pair and the stereoscopic plotter designed by Konshin, known as the multiplex. Measurements are taken from the model upon its completion.

Both of these variants are based on the conditions under which the photographing of the area was performed, with a single photograph or by an overlapping pair of aerial photographs.

Assume that, at the instant of photographing, the optical axis of the aerial camera was at an exactly vertical position, but that the surface of the area was at an angle differing considerably from the horizontal plane. Then, according to the rules of polyconic projection, the image of any point on the surface being photographed will be deviate, from the orthographic projection of the same point. As is known, the extent of such discrepancy is called relief distortion, which changes with the initial horizontal plane. In cases in which the original horizontal plane passes through the point on the surface being photographed, the orthographic projection of this point will coincide with its location in the area and the relief distortion is eliminated from the photograph. Due to the fact that, during the displacement of the initial horizontal plane, a change in the distance between it and the

center of projection takes place, the point on the ground can be made to coincide with its orthographic projection by changing the altitude of the aircraft or otherwise changing the scale of the photograph. Consequently, it can be assumed that all points on the surface, located in the same horizontal plane, will be reproduced on the photograph at a scale which is determined by the focal length of the camera and the distance from the center of projection to the known horizontal plane. Thus, the aerial photograph of a plane area will have a variable scale, whose value will change with any change in the elevation of the terrain according to the relation:

$$\frac{1}{m} = \frac{f_k}{H-h} \quad (93)$$

where $\frac{1}{m}$ is the scale of photography for a certain point of the photograph, which is the image of this point on the ground, at an altitude h relative to the initial horizontal plane. Because of this, in preparing a graphic plan from such photographs, the variation of the scale within the photograph must be taken into consideration, and the proper scale ratio must be calculated for each cross section, in accordance with the scale of the map. If the scale of the composite map is denoted by $1:M$ and if k denotes the scale ratio of map to photograph, the value of k will differ for each section and can be determined from the equation:

$$k = \frac{1}{M} \cdot \frac{1}{m} = \frac{m}{M} = \frac{h-h}{f_k M} \quad (94)$$

In practical application, changing the value of k for each section becomes a tedious problem, so that this is generally not done for each individual section but only for each area section, and under conditions where the difference between polyconic and orthogonal projection does not exceed the specified accuracy limits of the map. For example, if the value k_1 is used for preparing a graphic plan by sections, having the elevations h_1 and h_2 , then the errors to be expected are determined from the equation:

$$\Delta k = \frac{H-h_1}{f_k M} - \frac{H-h_2}{f_k M} = \frac{h_2-h_1}{f_k M} \quad (95)$$

$$\Delta l = l \Delta k \quad (96)$$

where Δl = Error on the map;

l = a section, being transferred to the map;

Δk = error produced by using the coefficient k_1 instead of k_2 in preparing the graphic map.

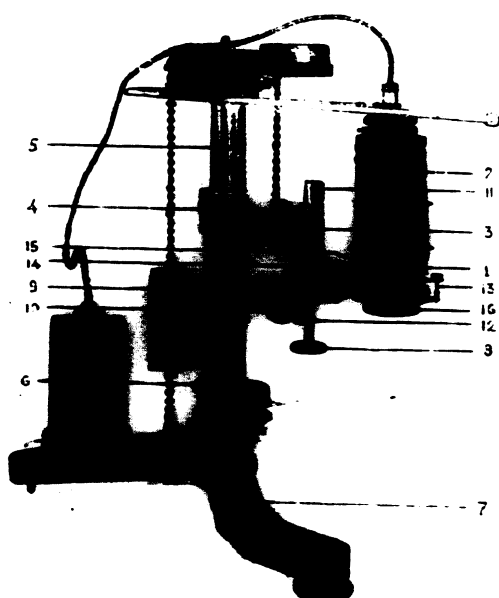
If the permissible error Δl is 0.5 mm, the segment l is equal to 50 mm, the scale for the graphic plan is 1:100,000, and $f_k = 70$ mm, then

$$\Delta k = \frac{0.5}{50} = \frac{1}{100}; \quad h_2 - h_1 = \frac{70 \times 100}{100} = 70 \text{ m}$$

Therefore, within the limits of 70 m it is permissible to use the constant value of k_1 . Thus, when marking contour lines of 70 m interval on the photograph, it is permissible to transpose directly from the photograph onto the map all contours and horizontals lying within the boundary of one section, which is defined by a constant reduction factor. For contours lying within the limits of another section, the reduction factor must be changed in accordance with the above equation. Such a change in the reduction factor and, consequently, the preparation of the map, is feasible by using various instruments. In actual practice, however, a strictly vertical position of the optical axis of the camera is never encountered, so that the aerial photograph is usually taken at a tilt. In such cases, in addition to changing the scale of the print to compensate for the slope of the terrain, further adjustment in values is required to compensate for the angle of tilt. Therefore, in preparing a graphic map, not only the reduction factor has to be changed for the various sections of the photograph, but the photograph must also be transformed. The instruments used in making the graphic map must be capable of solving both problems.

96. Making the Graphic Map with the Aid of a Projector (or Transforming Printer)

The main parts of the projector (Fig. 175) are the camera 1 and the attached light source 2. By means of a special mounting bracket 3, the camera is suspended from the horizontal track 4 which, in turn, is mounted to the vertical track 5. The



camera, together with the light source and the horizontal track, can be shifted vertically over the screw 6. The vertical tracks are mounted to the base 7 which has three leveling screws.

The camera has linear motion along the X, Y, and Z axes of the instrument. The motion along the X axis is accomplished by the bracket 3, along the horizontal track 4 over the screw 8. Motion along the Y axis is achieved by moving the horizontal pivot shaft 9 in its sleeve by means of the screw b_y - 10. Motion along the Z axis, for large ranges, is effected by

moving the horizontal track 4 along the vertical track 5 and, for small ranges, by moving the vertical shaft of the bracket 11 with the aid of the screw b_z - 12. The camera has three angular motions: the motion α_x in the XZ plane; the motion α in a plane perpendicular to it; and the motion χ in a rotary plane about its own axis. The camera is rotated through the angle α_x by turning the screw 14, through the angle α by turning the screw 13; and through the angle χ by turning the screw 15. The size of the projector camera is 6×6 cm. The focal length of the camera is

52.2 mm. The focal length can be adjusted for obtaining a sharply defined image when its scale is changed. This is done by screwing the lens in or out over the adjusting ring 16.

Making the graphic plan by sections has its peculiarities which are commonly divided into the following processes: a) preparation of the photographs and making negatives of them, b) transformation of the photograph and sectional plotting of contours and horizontals, c) fitting in the contours and horizontals.

The preparation of the photographs consists in selecting the sections for transformation, making corrections of the relief in relation to the transformation points on the base, bleaching out the image on the drawings, and preparing the negatives for the projector.

The photographic image is bleached out to make the maps readable as to drawn-in detail. Without bleaching, details of the terrain features (woods, roads, etc.) are lost in the underexposed and shaded portions of the photograph. The bleaching of photographs is accomplished in the same manner as in the case of photomaps.

The negatives of the bleached prints are produced by photographing them with the same projector as used for preparing the map. At this time, the print is reduced in size to the size of the projector plate holder.

In processing the negative, the aerial photograph is so placed on the screen that its nadir point coincides on the screen with the projection center of the contact frame of the projector camera. The frame itself must be horizontal at this time.

In selecting the sections for transformation, the interval between the sections is calculated from the height h' (Fig. 74) which yielded the maximum limit Δh of residual displacement of relief on the scale of the plan. In accordance with eqs. (12) and (25), the value of h' is determined from the equation

$$h' = 2 \Delta h \frac{f}{d_0} \quad (97)$$

where d_0 denotes the distance from the nadir point to the edge of the working sur-

face of the photograph, i.e., to the transformation point at the scale of the map: Δ_p limits are usually permissible to 0.4 mm.

Having determined the interval h' , the boundaries of the sections are determined and marked on the print, tracing the horizontals and defining the elevation of the mean planes T_i of the sections (Fig. 74).

The corrections for relief are calculated from eq. (25) and are applied to the positions of the points on the base. The new position of the points is marked with a sharp pencil, but without pricking. Corrections less than 0.5 mm are not made, but their direction is marked by an arrow, next to which the value for the correction is entered. During the transformation, they estimated by eye measure. Corrections for relief are applied to the position of transformation points only for the initial section, for the elevation difference of these points with respect to the mean plane T_0 of this initial section.

For projection, the negatives are placed into the projector, and the principal point (nadir point) of the print is aligned with the principal point of the projector camera, which is determined by observation on the screen. The print is then transformed from the transformation points of the base only for the mean plane T_0 of the initial section. After transformation in accordance with the projected image, the base composite is left on the plane table and the contour lines and horizontals are drawn in pencil within the boundary limits of this initial section.

For transferring the contours and horizontals to another adjacent section (as well as to all following zones), the scale of the image is varied, without changing the tilt of the photograph. On passing from one section to the other, the scale is changed by varying the height of the projector above the plane table by the quantity ΔZ , which is determined by the equation

$$\Delta Z = Z \frac{h'}{h} \quad (98)$$

where Z denotes the height of the projector above the screen during the transformation of the print to the mean plane of the first section; h' is the height difference

in passing from one section to the other; H' is the height of the camera station above the mean plane of the first section.

The change in scale, when going from one section to the next, can be controlled by the distance between two distant points on the print (e.g., transformation points). This distance is plotted on paper (or steacil) and the corrections for relief are constructed for its end points when passing from the mean plane T_0 of the initial section to the mean plane T of all other sections of the photograph. For example, let $h' = 30$ m, the height of the initial plane for the second section $T_0 = 200$ m, and the height of the mean plane of the first, third, and fourth section be $T_1 = 170$ m, $T_3 = 230$ m, $T_4 = 260$ m. Then the correction for relief variation for the two transformation points x_1, x_2 having the elevation $h_1 = 180$ m and $h_2 = 230$ m are calculated from their elevation differences.

For the initial section $h^0_1 = -20$ m; $h^0_2 = +30$ m

For the first section $h^1_1 = +10$ m; $h^1_2 = +60$ "

For the third section $h^3_1 = -50$ m; $h^3_2 = 0$ "

For the fourth section $h^4_1 = -80$ m; $h^4_2 = -30$ "

These corrections for the points x_1 and x_2 are laid off on a strip of paper; when adjusted to the scale for the first section, the images of the points x_1 and x_2 of the photograph must coincide with their location on the paper. adjusted after the corrections for elevation difference $h^1_1 = +10$ m and $h^1_2 = +60$ m; for the third zone from points displaced by the elevation distance of $h^3_1 = -50$ m, $h^3_2 = 0$ etc. have been made. After adjusting the scale for each successive section, the contours and horizontals are transferred from the photograph to the map within the boundary limits of each separate section (see Fig. 74). Prior to this, the tracing table is oriented by points and contours of the image for each of the sections (except for the initial zone). The need for such orientation arises from the fact that, in changing the height of the projector, the image points are displaced in proportion to their distances from the perpendicular to the base plane, intersecting the lens

center of the projector. On tilting the projector the nadir point is displaced from this vertical so that, besides a change in the scale, there will be a general shift of the image on the tracing table. Because of this, after changing the scale, the tracing table is moved to realign the projected image of the nadir point with its position marked on the table, and the table is rotated in such a way that the radials from the nadir point to the control points (transformation) coincide with those of the image.

During the transformation of the photograph by means of the projector, it must be borne in mind that the projector has no devices for maintaining the conditions of correct transformation, and that a strictly accurate transformation is possible only if the projector is used as transforming printer of category I.

In processing photographs with poor similarity of the elements of interior orientation, certain distortions in the image will result, which do not become evident at low angles of tilt, such as $1^{\circ}5'$. However, at high angles of tilt, the points on the tracing table will not coincide with those of the image so that a graphic map with this type photographs must be prepared under identical conditions on a transforming printer. In many cases it becomes advisable to first transform the photographs with large angle of tilt and then to process them with a projector by the above described method.

After completing the transfer of the situation and relief from the photograph to the map, the transferred elements are recorded and the accuracy and completeness of transfer is checked against the photographs. In addition, the margins and kilometer grid of the trapezoid are checked. For final reproduction, a blueprint is made of the corrected plotting board.

It is recommended to prepare the map at a scale slightly larger than that of the final map, at a coefficient of enlargement of 1.2-1.3, to level out the errors of computation and drafting during subsequent reduction of scale.

97. Making a Graphic Plan with a Stereoscopic Plotter

The stereoscopic plotter, illustrated in Fig. 176, is actually a stereoscope permitting a simultaneous viewing of the stereo model and the plane table. The stereoscope is located underneath the plotter itself, which allows the tracing of the graphic plan simultaneously with the stereoscopic plotting of the relief.



Fig. 176 - Stereoscopic Plotter

To the base 1 of the instrument the vertical tracks 2 are mounted, along which the carrier 4 is displaced by turning the hand wheel 3. The carrier contains the photograph holders 5 as well as the optical system of the instrument.

The optical system (Fig. 176 and 177) consists of a mirror 6 and cubes 7. Each of these cubes is made up of two prisms, cemented together along the diagonal plane. Half of this diagonal plane is silvered and thus simultaneously permits passage of light beams coming from the points on the plane table, and reflection of light beams coming from the photograph, to reach the eye of the observer. Due to this, the observer sees both photograph and plan at the same instant.

The cubes 7 and the right mirror are rigidly attached to the housing 4. The left mirror 8 and the left print holder are attached to a secondary carrier moved by the screw 9 along the main carrier parallel to the eye base. This shifts the left mirror along its own parallel plane, together with the print, producing an elongation of the light beam and a reduction of the field of view of the image. This compensates for the scale difference, due to photographing at various altitudes.

Any print placed in the instrument can be adjusted in any direction to orient it in space (external orientation), i.e., by rotating it through the angle α_{x_1} by means of the screw 10, about the horizontal axis perpendicular to the eye base. The rotation ω takes place about the axis perpendicular to the axis of rotation α_x , and is accomplished by the screw 11. The rotation of the print in its own plane, about an axis perpendicular to the axes of rotation α_x and ω is accomplished over the screw 12.

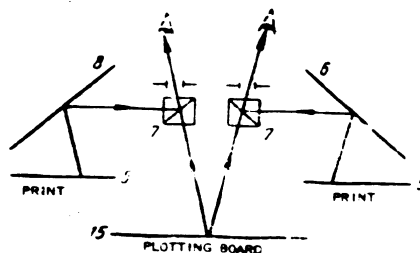


Fig. 177 - Schematic Diagram of the Stereoscopic Plotter

The displacement of the print in its plane along the xx axis (for a horizontal print, parallel to the eye base) is accomplished by the screw 13, and along the yy axis by the screw 14. The extent of all of these displacements is registered on the corresponding instrument scales.

The displacement of the carrier 4, which moves the optical system and prints along the tracks 2, relative to the plotting board 15 produces a noticeable change in scale on the plotting board, thus providing a means of adjusting the viewed image of the print to the same scale as that on the plotting board. In addition, this motion permits measuring the stereo model by means of intersecting its surface at the plane at various elevations. A dial indicator is provided for measuring this motion along the vertical track. The viewing is done through a set of binoculars 16, fitted on top of the instrument which also provides a comfortable viewing

1

position for the observer's head.

Preparation of a graphic map by means of the stereoscopic plotter falls into two main categories: orientation of the prints relative to the control points on the plan and actual direct process of producing the plan.

Orientation of the prints is accomplished by transforming each print separately in accordance with the control points (transformation points) of the plotting board. Consequently, corrections are applied to the control points to compensate for relief deviation from the true initial transformation plane. The transformation of prints by the stereoscopic plotting method is done in accordance with second-order transformation, which makes it possible to impart to the prints all angular motions and linear eccentricing (which, e.g., is impossible with projectors).

The plan is prepared while viewing of the stereo model, following an individual transformation of the left and right photographs with respect to their common control points, at which time the scale of the developed model can be changed by rotating the cube and the print holder about an axis perpendicular to the eye base. Rotating the cube will change the direction of the projected ray directed to the points on the print, while rotating the print holder through twice the angle will compensate for the perspective distortions which arise in this case.

The process of making the plan in accordance with the developed stereo model of the area is as follows:

Changing the height of the carrier 4, together with the prints and the optical system, will change the height of the entire stereo model, but will not affect or change the orientation with respect to the established plane of transformation. This results in an intersection of the surface of the stereo model on the plotting board, at different elevations (Fig. 178).

Points a, b, c. (position 1, Fig. 178) represent the intersection points of the stereo model with the plotting board and represent the orthographic projection of these points onto the board. Other points on the model, such as k, m, e, n, will

be seen higher than the level of the plotting board, while point d will be lower than this plane (or the contour images of the right and left print of this point will appear separated).

While in position I all the points plotted on the photographs, such as contour lines and horizontals lying in the intersection plane with the plotting board, i.e.,

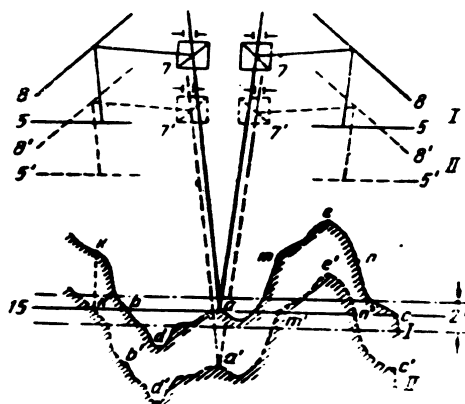


Fig. 17B - Measuring the Model with the Stereoscopic Plotter

coinciding with this board, plotting board. These will be points on the stereo model located in the same horizontal plane at equal elevations. For transferring the contours and horizontals located at different elevations, e.g., points k, m, n the elevation of the carrier 4 is changed until these points coincide in elevation with the plane of the plotting board (position II, points k', m', n'), a method suitable for stereoscopy. Having achieved coinci-

gence, the contours and horizontals for this new elevation of the stereo model are plotted and transferred to the plotting board.

However, if in establishing each position of the stereo model, only those points are transferred which lie strictly in the plane of the plotting board, an excessive number of adjustments would be required for completing the entire model. This would render the process very complex. Consequently, transfer of points is made not only for the points on the stereo model that intersect geometrically with the plane of the plotting board but also for points lying in the vicinity of the plotting board and having small values of h , above the level of the board which

limits the number of total points. These arbitrary points are selected in such a way that their error would not exceed the specified limit of map accuracy. In this way, for each position of the stereo model (I, II, etc.) the approximate height interval $2h$ is transferred to the plotting board (Fig. 178), which gives an approximate strip of the stereo model. Since transition from one elevation to another is very simple with this instrument, caution must be exercised not to enlarge the height interval $2h$ and thus to introduce excessive errors. By keeping the height interval smaller than permitted, the accuracy of the over-all plan is increased.

If the relief is drawn in directly by means of the stereoscopic plotter, while only the results of photographic interpretation are entered on the prints, then the model can be developed under the assumption that the positions of the tracings on the plotting board would correspond to those of the horizontals. Then the lines, denoting the points of the model which coincide exactly with the plane of the plotting board can be considered the contours, which will be drawn into the map. Simultaneously, the outlines lying within the acceptable limits of the contours are also drawn in.

The required elevation of the sectional area, in graduations of the vertical circle 2, can be determined from the elevation reference points. Let us assume that the geodetic elevation of the points a and b (Fig. 178) is known. After superimposing point a of the stereo model on the plane of the plotting board (position I), the elevation of the model is recorded with respect to its height in the instrument, as shown by the graduated scale of the vertical circle 2 (Fig. 179). Point b is then superimposed on the plotting board, and a second recording is made of the new reading on the scale. Dividing the difference between the two geodetic elevations by the difference between the two scale readings, the value per dial graduation for the elevations of the stereo model at the specified scale is obtained. Knowing the readings of the dial for each of the two corresponding points of known geodetic elevation, as well as the value per scale graduation, the dial readings corresponding

to the sectional contours can be obtained by interpolation (and extrapolation).

After completion of the plan, the pin pricks are all connected by lines to form contours and horizontals of the sections of the stereo pairs. The sectional lines are also interconnected, as are the stereo pair lines. The work is thoroughly checked for completeness and correctness of the drawing.

CHAPTER XII

CONSTRUCTION OF A THREE-DIMENSIONAL MODEL

98. The Double Projector

In aerial stereophotogrammetric surveying, the double projection method is in widespread use. This method duplicates the conditions prevailing while the aerial camera work was being conducted. The double projector consists of a rigid base 1 (Fig. 179), two projection cameras 2, and a screen 3. Two adjacent aerial negatives (or glass-plate copies of the negatives) are placed in the projection cameras. The distance along a perpendicular dropped from the center (rear nodal point) of the projector lens to the plane of the positioned negative must be equal to the focal length of the aerial survey camera; the principal point of each photograph should coincide with the foot of this perpendicular. Negative illuminators are placed above the projection cameras. This permits a photographic image to be obtained on the screen. The screen becomes the plane of the terrain and can be brought nearer to the projection cameras or moved farther away from them. Each projection camera can move, relative to the fixed screen, along three mutually perpendicular axes, two of which are parallel to the plane of the screen, while the third is perpendicular to the plane of the screen. In addition, each projection camera can be inclined along two axes, and the negatives, mounted in the cameras, can be rotated in their planes.

Thus, the negative in each projection camera can be adjusted relative to the plane of the screen in accordance with six elements of exterior orientation, $X_s, Y_s,$

Z_S , α_x , α_y , and λ .

99. The Ground Model and its Scale

If two adjacent negatives are set in the projection chambers of a double projector, illuminated with a light source and are given, relative to the screen, the

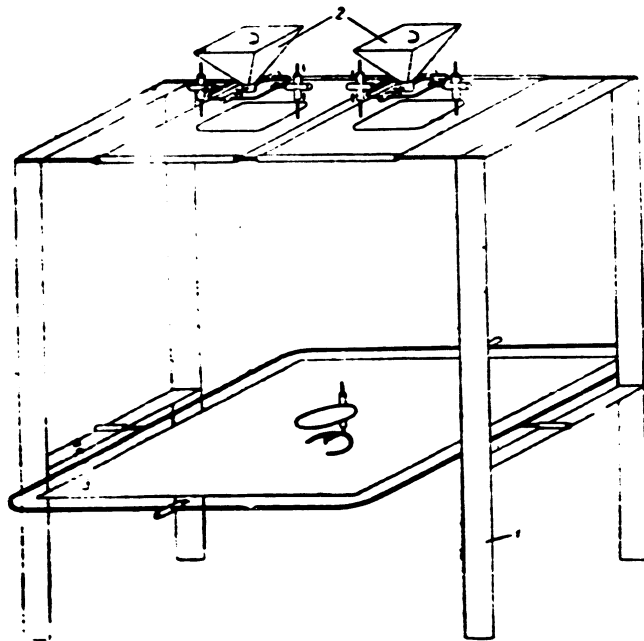


Fig. 179 - Schematic diagram of a Double Projector

same position in space which they occupied relative to the level surface of the ground at the instant of exposure, the projecting rays, passing through the projector lenses will travel in the same direction as when the photographs were made. In a given case, the process will be the reverse of the aerial photographic process: formerly, the projecting rays traveled from points on the ground to the photographs.

whereas now they are directed away from the points on the photograph. It is obvious that the rays, proceeding from identical points on the two photographs, will in their course, intersect at points coinciding with points on the ground. The combination of all of the points of intersection of identical rays forms a surface similar to the

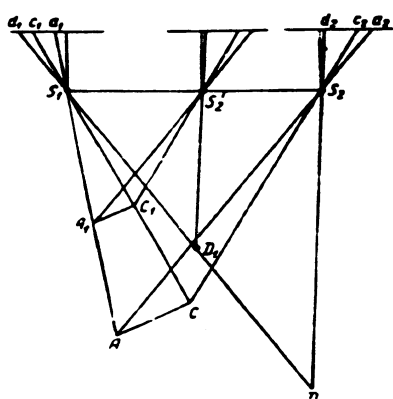


Fig. 180 - Scale of the Model

ground, which is called the ground model. Measurements on the ground model can, therefore, replace the corresponding measurements in the field. Since the construction of each point of the ground model occurs as the result of the intersection of two projecting rays emanating from centers of projection, this construction can be considered as the intersection from the ends of the photographic base, which is the distance between

the two centers of projection.

The constructed model might have the same dimensions as the area photographed, but the practical realization of such a construction is impossible by virtue of the fact that at the instant of exposure of the aerial photograph, the distances from the first center of projection to the second center of projection are equal to several kilometers. For this reason, the size of the base (projection base) on the double projector will be considerably less than on the aerial photograph itself. However, regardless of this reduction of the photographic base, the projecting rays, preserving their former directions, will intersect. In reality, if the projection base $B = S_1S_2$ (Fig. 180) is equal to the photographic base, the projecting rays, emanating from the corresponding points a_1 and a_2 , c_1 and c_2 , d_1 and d_2 of

the two photographs, will intersect at the points A, C, and E, of the model. On reducing the projection base, when the second center of projection is located at the S_2 position, the intersection of the projecting rays will give the points A_1C_1 and D_1 of the model. From the similarity of the triangles S_1S_2A and $S_1S_2'A_1$, S_1S_2C and $S_1S_2'C_1$, it follows that

$$\frac{S_1S_2'}{S_1S_2} = \frac{S_1A_1}{S_1A} = \frac{S_1C_1}{S_1C} = \frac{S_2A_1}{S_2A} = \frac{S_2'C_1}{S_2C}$$

and, accordingly, the triangles $S_1A_1C_1$ and S_1AC , as well as $S_2A_1C_1$ and S_2AC will also be similar. Hence,

$$\frac{S_1S_2'}{S_1S_2} = \frac{S_1A_1}{S_1A} = \frac{A_1C_1}{AC}$$

Since the ratio of the distances A_1C_1 and AC represents the scale of the area in question, then

$$\frac{S_1S_2'}{S_1S_2} = \frac{A_1C_1}{AC} = \frac{1}{m} \quad (99)$$

It is obvious from eq. (99) that the scale depends only on the ratio of the projection base to the photographic base and is a constant quantity for any distances measured on the surface of the model. Hence, the ratio of the projection base to the photographic base is called the scale of the model.

100. Measurement of the Model

A model constructed on a double projector can be measured. This measurement is made with the aid of a screen, shifted in the direction of the projecting lenses while maintaining a mutually parallel position. With the apparatus in a vertical position, the screen is horizontal and is moved in elevation for each measurement of the model.

If the screen is located below the point A of intersection of the projecting rays S_1a_1 and S_2a_2 (Fig. 181), two images A_1 and A_2 of identical points in the two photographs are obtained on the screen. This indicates that the point of the model does not lie in the plane of the screen. Conversely, the presence on the screen of a single image C of the corresponding points c_1 and c_2 in the two photographs permits assuming that the intersection of the projecting rays coincides with the plane of the

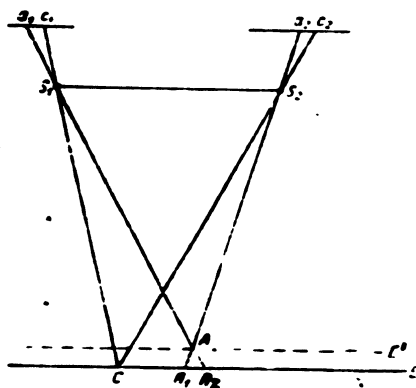


Fig. 181 - Measurement of the Model

screen. Therefore, the screen of the double projector can be regarded as a plane cutting the ground model at those points which give a single image; a curve connecting all points which show up as a single image will be a contour line of the corresponding section.

To obtain the plane of another section, the screen must be moved up or down, e.g., to the position E' . Then the projecting rays S_1a_1 and S_2a_2 will show up as a single image in the plane of the screen while the rays S_1c_1 and S_2c_2 will

show two images since the point A of the model now coincides with the plane of the screen, instead of point C of the model, as in the first position. Hence, the amount which the screen was displaced vertically is thus equal to the elevation difference of point A above point C, expressed in the scale of the constructed model. This vertical interval is measured with the help of a scale which registers the vertical movement of the screen. It is thus possible to move the screen up or down by the altitude of the section of the horizontal base and, by marking on the screen the position of all noncoincident points, to draw the contour line of the corresponding section. It is quite obvious that the distance between corresponding points on the

screen corresponds to the horizontal parallax difference on the photographs.

In duplicate projectors, the construction of the contour plan proceeds simultaneously with the plotting of the relief. Since orthographic projection is used in constructing the ground plan, and since polyconic and orthographic projections of any point are mutually coincident in cases in which the selected horizontal plane

passes through this point, the principle just described is used in constructing a map on duplicate projectors. Hence, if a single image of the corresponding points of two photographs is obtained on the screen of the duplicate projector, then, by marking its position on the screen the map position of the corresponding terrain point can be constructed. In this



Fig. 182 - Tracing Table for the Duplicate Projector

manner, the screen is marked with the plan position of contours situated close to the horizontal plane of the section occupied by the screen during the work.

Instead of moving the screen vertically and tracing the contour lines and terrain features on it by hand with a pencil, the special stand, shown in Fig. 182 is used.

To the horseshoe-shaped base 1 are fastened two standards 2, along which a small circular screen can be moved with the help of a screw 3. The center of the small screen is provided with a point aperture 4, illuminated from below by a small bulb. On the perpendicular dropped from the point aperture to the base of the stand, a pencil 5 is installed, which is raised or lowered to make contact with the paper below the stand. The tracing table is mounted to the plotting board on the screen of the duplicate projector. The projected images of corresponding points of the two

photographs show on the tracing table screen where they are viewed by the observer. Points of the model coinciding in elevation with the plane of the small screen are successively superimposed on the marker light spot and the trace obtained during this displacement of the stand is drawn by the pencil on the plotting board. This trace will be horizontal to the corresponding section. In changing the elevation of the small screen (by turning the Screw 3), the other points of the model will be superimposed on the plane of the screen, permitting the horizontal of another section to be drawn. The amount of vertical displacement of the screen is read on the scale in millimeters; this quantity is converted to meters by multiplying it by the denominator of the scale of the model.

To draw the contour lines of the photographed area on the plotting board, the light spot of the stand is successively superimposed on the various points of the projected contour, which at the moment of coinciding with the light spot will throw only a single image on the small screen. If various points of this contour have different elevations, the elevation of the screen must be changed in drawing the contours. This will cause the tracing pencil of the stand to plot an orthographic projection of the given contour.

Thus, to measure a constructed model a tracing stand is used whose light spot serves as a reference mark whose motion is matched by the pencil. The model is measured by making the light spot coincide with the ground forms while continually varying the elevation of the screen. Contour lines are traced by continuous matching of the light spot with points on the image which do not diverge, keeping the top of the tracing stand at a constant elevation.

101. Study of the Model

In measuring a model to determine relative vertical intervals of separate points and to draw contour lines, accurate recording of the changes in elevation of the top of the tracing stand is required. For example, a change in elevation of the

points equal to 1 m, at a model scale of 1:10,000 is expressed as 0.1 mm which, at an angle of intersection of 45° , corresponds to a division of 0.1 mm for the images of corresponding points. Hence, separation of images on the top of the tracing stand must be established with a high degree of accuracy.

Various methods of observation are used to increase the accuracy of measurements on duplicate projectors. The most common of these methods is the anaglyphic method, based on the property of complementary colors to give off white light when combined. Blue-green and red are used when working with this apparatus.

If a red filter is placed over the lens of the left camera of the duplicate projector, the image projected will also be red. An observer viewing this image through a red glass will perceive the entire image. At the same time, if the observer views the image through a blue-green glass, he will not perceive the image since the blue-green glass will not transmit red rays. Similarly, a blue-green image projected onto the screen by a blue-green filter can be seen through blue-green glass, but not through a red glass. If two filters, red on the left and blue-green on the right lens, are used at the same time and the images are viewed through a red glass over the left eye and a blue-green glass over the right eye, each eye will perceive only one image. Correspondingly, the viewing conditions will be like those encountered in a stereoscope, with the observer perceiving a three-dimensional image of the area photographed. This spatial model will be cut by the plane of the screen, permitting a considerably more accurate determination of the moment of coincidence of the plane of the screen with points on the model.

The principle of the anaglyphic method of observation has been widely used in creating special anaglyphic charts and drawings. In the case in question, a sheet of paper is placed on the plane of the screen. The paper carries an image printed in red, corresponding to one projection camera, while the second image is printed in green. Since the points of the model located higher or lower than the plane of the screen have differing values of separation (horizontal parallax differential), all

points on one contour line are given a single separation value in preparing the anaglyphic charts (uniform horizontal parallax difference) which differs from the horizontal parallax differential of other points. Thus the image printed in green will match the red image only along one contour line and will not coincide along other contour lines, but will differ by varying quantities. In viewing such a chart through glasses with one green and one red lens, the observer will see the green image through the red lens and the red image through the green lens. When both eyes are used at the same time, the two images resolve into one spatial picture and the observer sees a relief image of the corresponding portion of the chart.

The difference between perception of the images printed on paper in two colors and those projected through filters onto the screen consists in the fact that through the red lens, in the former case, the observer sees the green image and in the latter case he sees the red image. This difference is due to the fact that the red filter passes only the red rays of the spectrum, which are transmitted further by the red lens of the glasses but not by the green lens. In the former case, the red dye with which the image is printed on the paper absorbs (not reflects, as in the second case) the red light and, during observation through red glass, the image combines with the background; whereas when viewed through green glass it appears black, standing out from the background.

There are methods other than the anaglyphic which are used in double projection. However, those are not as widely used.

Thus, in measuring models on a duplicate projector, the stereoscopic principles of viewing is used, employing anaglyphic filters based on the relationship of complementary colors. Two optical filters tinted in two complementary colors are used one for each projection camera. Observation of the colored images projected on the screen is accomplished with the help of glasses whose lenses are tinted in the same complementary colors. In viewing with both eyes through the glasses, the observer sees a stereoscopic model of the terrain, cut at various points by the plane of the

screen as the elevation of this screen is varied. The path cut by the screen through a section of the model is a contour line of the corresponding section.

102. Relative Orientation of the Photographs

The area model constructed on the duplicate projector comprises a unit with two projection cameras and does not depend on the position of the instrument screen. Hence, if the two projection cameras are shifted simultaneously and linearly along the three axes of the space coordinate, or are rotated together about the same axes,

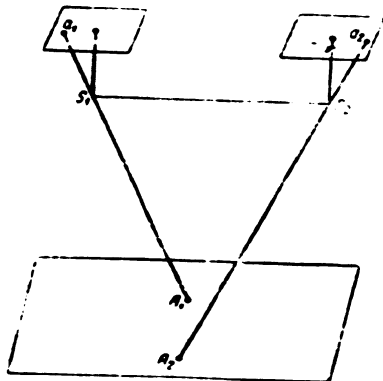


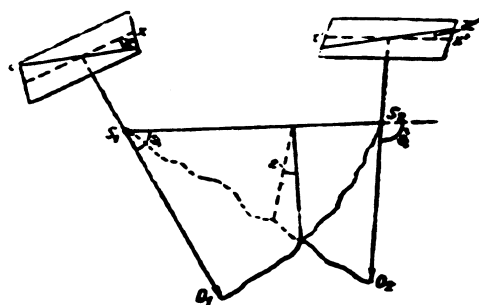
Fig. 183 - Intersection of Projecting Rays

the position of the model relative to the plane of the screen will change while the intersection of the corresponding projecting rays will be retained. Thus, a simultaneous change in the elements of exterior orientation of the two cameras changes only the position of the model relative to the screen, while the model itself remains unchanged. The situation is different in the case when one of the projection cameras retains its elements of exterior orientation while these elements are changed for the other camera. Thus, e.g., if only the right camera is rotated about the base line, the projecting ray S_2a_2 (Fig. 183) will not lie in a plane with the ray S_1a_1 . Consequently, these rays fail to intersect, in which case, as we know, no model of the terrain can be obtained. This statement applies to other displacements of only one of the cameras (independent of the other), except when a camera is moved along the base line, in which case only the scale of the model changes.

the position of the model relative to the plane of the screen will change while the intersection of the corresponding projecting rays will be retained. Thus, a simultaneous change in the elements of exterior orientation of the two cameras changes only the position of the model relative to the screen, while the model itself remains unchanged. The situation is different in the case when one of the projection cameras retains its

It is clear from the foregoing that an area model is constructed only when the projection cameras, relative to each other, occupy the same position as at the instant of exposure, i.e., when their relative position is maintained. Any disturbance in the relative position of the two cameras results in failure of the corresponding projecting rays to intersect, so that no model of the terrain is produced.

The relative position of the two photographs is determined by the elements of relative orientation, which can be represented by various methods. If we imagine



that the plane passing through the base line and the optical axis of the left photograph (the so-called principal base plane of the left photographs) is vertical and coincident with the XZ plane of the system of space coordinates, while the base line is horizontal, then the elements of relative orientation are determined by the quantities ϕ , ϕ' , ϵ , γ , and γ' (Fig. 148). Here

Fig. 184 - Elements of Relative Orientation

ϕ denotes the angle formed in the principal base plane of the left photograph by the direction of the optical axis with the base line; ϕ' is the angle formed in the principal base plane of the right photograph by the direction of the optical axis with the base line; ϵ is the angle between the principal base planes of the two photographs; γ is the angle formed in the plane of the left photograph by the trace of the principal base plane and the direction taken for the xx axis of the photograph; and γ' is the angle formed in the plane of the right photograph by the trace of the principal base plane and the direction taken for the $x'x'$ axis of the photograph.

Thus, the elements of relative orientation constitute five quantities, since any rotation, tilt, or linear shift of the base line will not disturb the intersec-

tion of the corresponding rays; at the same time, any change in the elements of relative orientation results in failure of the projecting rays to intersect. Actually, if a plane (called the base plane) is drawn through any point of the model and any base line, then any change in the elements of relative orientation will cause the corresponding projecting rays to be located in different base planes that do not coincide. The angle formed by the two base planes containing the projecting rays from the corresponding points of the two photographs is called the angular parallax. The presence of angular parallax indicates that the projecting rays from corresponding points of the two photographs do not intersect and do not form a model of terrain. By setting the screen in the paths of the projecting rays parallel to the base line, and by taking the trace of the base plane as the direction of the xx axis, it is simple to note the doubling of projections of corresponding points in the direction of the yy axis, perpendicular to the xx axis. This doubling is the result of angular parallax and is known as linear or horizontal parallax, usually designated by the letter q . Therefore, as was shown in Section 81, the angular parallax is equal to the difference in the ordinates of the projections of corresponding points of the two photographs, i.e.,

$$q = y - y'$$

Thus, the presence of linear parallax indicates that, in the duplicate projector, the relative position of the photographs is inaccurately adjusted since, at correct adjustment, there can be no linear parallax at any point of the model.

Inaccurately adjusted elements of relative orientation lead to different values in the X -parallax depending on the coordinates of the corresponding points. For example, if, after correct relative orientation, one of the photographs is rotated in its plane through the angle χ , the principal point of the given photograph will show no X -parallax; in all other points, the X -parallax will be proportional to their abscissas. For this reason, it is possible to formulate a mathematical relationship

between the magnitude of X-parallax and the elements of relative orientation, whose coefficients will be the current coordinates of the observed points. At low angles of tilt of the optical axis, and negligible rotations of the photographs in their planes, the relationship between the X-parallax and the elements of relative orientation is expressed by*

$$q = -\frac{xy}{f_k \rho} \tau + \frac{x'y'}{f_k \rho} \tau' + \frac{f_k^2 + y^2}{f_k \rho} \epsilon + \frac{x}{\rho} \chi - \frac{x'}{\rho} \chi' \quad (100)$$

where q is the X-parallax of any point, x , y and x' are the current coordinates of this point on the left and on the right photographs, and τ and τ' represent the differences in the angles φ and φ' from 90° , i.e., $\tau = 90^\circ - \varphi$ and $\tau' = 90^\circ - \varphi'$.

103. Determining the Elements of Relative Orientation

Since the magnitudes of the X-parallax and the current coordinates of a selected point can be measured, eq. (100) is an equation with five unknowns, which are the elements of relative orientation. Therefore, to determine the elements of relative orientation, five analogous equations must be constructed, and the solution of the resultant system of equations must be used for determining the values of the unknowns. Then, to determine the elements of relative orientation, five points within the limits of the stereo pair are selected (six are generally used to facilitate control of the solution) and measured for their current coordinates and X-parallax; the solution of this system of equations will yield the basic data for adjusting the projecting cameras of the duplicate projector.

In selecting the points whose X-parallaxes are to be measured, three conditions must be observed: 1) the selected points must permit determination of the elements of relative orientation with the highest degree of accuracy; 2) the equations obtained must be independent of each other; 3) the equations must be in the simplest

* See Section 129 for derivation of the equation

possible form. Considering eq. (100) from these points of view, we can easily see that if $\tau' = \varepsilon = \chi = \chi' = 0$

$$\Delta q = - \frac{xy}{f_k \rho} \Delta \tau$$

where Δq is the error of measurement of the X-parallaxes, while $\Delta \tau$ is the error of determination of an element of relative orientation. Hence,

$$\Delta \tau = - \frac{\Delta q f_k \rho}{xy} \quad (101)$$

Equation (101) shows that a constant error in the measurement of X-parallax has the smallest effect on the angle τ at maximum values of the ordinate and abscissa of the observed point of the left photograph. Similarly, using successively

$$\begin{aligned} \tau = \varepsilon = \chi = \chi' = 0; \quad \tau' = \tau = \chi = \chi' = 0; \quad \tau' = \tau = \varepsilon = \chi' = 0 \\ \tau' = \tau = \varepsilon = \chi = 0 \end{aligned}$$

it is possible to obtain:

$$\begin{aligned} \Delta \tau' = + \frac{\Delta q f_k \rho}{x'y} \quad \Delta \varepsilon = + \frac{\Delta q f_k \rho}{f_k^2 + y^2} \\ \Delta \chi = + \frac{\Delta q \rho}{x} \quad \Delta \chi' = \frac{\Delta q \rho}{x'} \end{aligned} \quad (102)$$

Consequently, at a constant error in the measurement of the X-parallax, the angle τ' is obtained with the least error at the largest value of the ordinate and abscissa of the observed point of the right photograph, the angle ε at the largest value of the ordinate of the observed point; the angle χ at the largest value of the abscissa of the observed point of the left photograph, and the angle χ' at the maximum value of the abscissa of the observed point of the right photograph. Accordingly, to determine the elements of relative orientation with the greatest accuracy,

the selected points must be located on the photograph in the zones indicated in Fig. 185. This distribution of the points also satisfies the requirement that the equations be independent and that their solution be in the simplest possible form.

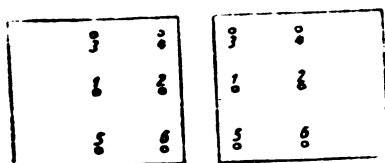


Fig. 185 - Distribution of Points for Relative Orientation

Taking the horizontal parallax difference of the points as equal to zero (case of flat terrain where when $x' = x - b$) we obtain the following values for the current coordinates of the selected points:

$$x_1 = 0; y_1 = 0; x'_1 = -b; x_2 = b; y_2 = 0;$$

$$x'_2 = 0; x_3 = 0; y_3 = y$$

$$x'_3 = -b; x_4 = b; y_4 = y$$

$$x'_4 = 0; x_5 = 0; y_5 = -y; x'_5 = -b; x_6 = b; y_6 = -y; x'_6 = 0$$

With the indicated values of the current coordinates, the equations of relative orientation will take the form:

$$\left. \begin{aligned} q_1 &= + f_k \frac{e}{\rho} + \frac{b}{\rho} \lambda' \\ q_2 &= + f_k \frac{e}{\rho} + \frac{b}{\rho} \lambda \\ q_3 &= - \frac{by}{f_k \rho} \tau' + \frac{f_k^2 + v^2}{f_k \rho} e + \frac{b}{\rho} \lambda' \\ q_4 &= - \frac{by}{f_k \rho} \tau + \frac{f_k^2 + v^2}{f_k \rho} e + \frac{b}{\rho} \lambda \\ q_5 &= + \frac{by}{f_k \rho} \tau' + \frac{f_k^2 + v^2}{f_k \rho} e + \frac{b}{\rho} \lambda' \\ q_6 &= - \frac{by}{f_k \rho} \tau + \frac{f_k^2 + v^2}{f_k \rho} e + \frac{b}{\rho} \lambda \end{aligned} \right\} \quad (103)$$

Subtracting the fifth equation from the third and the sixth from the fourth will yield

$$q_3 - q_5 = - \frac{2 by}{f_k \rho} \tau'; \quad q_4 - q_6 = - \frac{2 by}{f_k \rho} \tau$$

from which

$$\tau' = - \frac{(q_3 - q_5) f_k}{2 by} \rho; \quad \tau = - \frac{(q_4 - q_6) f_k}{2 by} \rho \quad (104)$$

The difference in the angles τ is the difference in the longitudinal angles of tilt of the two photographs; hence,

$$\Delta \alpha_v = \tau - \tau' = - \frac{(q_4 - q_6 - q_3 + q_5) f_k}{2 by} \rho \quad (105)$$

To determine the other unknowns, the third and fifth equations must be added and twice the first equation subtracted from their sum. Then,

$$q_3 + q_5 - 2 q_1 = + 2 \frac{f_k^2 + y^2}{f_k \rho} \epsilon + 2 \frac{b}{\rho} \chi' - 2 \frac{f_k}{\rho} \epsilon - 2 \frac{b}{\rho} \chi' + \frac{2 y^2}{f_k \rho} \epsilon$$

or

$$\epsilon = + \frac{(q_3 + q_5 - 2 q_1) f_k}{2 y^2} \rho \quad (106)$$

By analogy, adding the fourth and the sixth equations and subtracting twice the second equation from their sum, we obtain

$$q_4 + q_6 - 2 q_2 = + 2 \frac{f_k^2 + y^2}{f_k \rho} \epsilon + 2 \frac{b}{\rho} \chi - 2 \frac{f_k}{\rho} \epsilon - 2 \frac{b}{\rho} \chi + \frac{2 y^2}{f_k \rho} \epsilon$$

or

$$\varepsilon = + \frac{(q_4 + q_6 - 2 q_2) f_k}{2 y^2} \rho \quad (106')$$

Thus, the angle ε will be obtained twice in two independent processes, constituting a check on the correctness of the measurements and calculations made. Finally, the last two elements of relative orientation are found from the first two equations:

$$\chi = - \frac{f_k}{b} \varepsilon + \frac{q_2}{b} \rho \quad (107)$$

$$\chi' = - \frac{f_k}{b} \varepsilon + \frac{q_1}{b} \rho \quad (108)$$

Thus, the problem of determining the elements of relative orientation consists in measuring the X-parallax at six points arranged according to a given standard, and in calculating the unknown quantities according to eqs. (104) to (108).

104. Determination of the Elements of Relative Orientation with a Stereocomparator

In the solution of a number of photogrammetric problems, particularly in the orientation of photographs on a stereometer, it is first of all necessary to know the elements of relative orientation. These quantities are usually found by solving the corresponding equations for the X-parallax, measured on a stereocomparator, of a standard array of the six points whose coordinates are shown in Table 8.

To measure the transverse parallaxes, adjacent negatives are placed on the carriers of the instrument, as indicated in Section 83, and are oriented according to the initial direction. Following this, the left reference mark is then made to coincide with the principal point of the left photograph, while the right photograph, by means of the horizontal parallax screw, is placed in a position close to the corresponding point of the right photograph at which, however, the two marks are

not yet merged into single floating mark. Then, viewing the photographs with both eyes, the observer sees that one mark is displaced relative to the other along the yy axis, and corrects this displacement by turning the transverse parallax screw.

Table 8

Coordinates of Points of the Left Photograph in the Determination of the Elements of Relative Orientation

Nos. Points Coordinates	1	2	3	4	5	6
x	0	+ b	0	+ b	0	+ b
y	0	0	+ y	+ y	- y	- y

Then, by turning the horizontal parallax screw, the two marks are made to merge into a single marker which is set tangent with a point of the model corresponding to the principal point of the left photograph. When this coincidence is obtained, readings are taken on the x, y, P, and Q scales of the instrument and are recorded in the corresponding record. In the same manner, the floating mark is made to coincide with a point of the model corresponding to the principal point of the right photograph, and readings are again taken on the scales of the instrument; the y-scale reading remains unchanged if the orientation according to the initial direction was carried out correctly.

To match the floating mark with the third selected point, the x scale is set to the reading taken when the floating mark and the initial point were aligned, while the y scale is set to the former reading, increased by a constant quantity (60 or 70 mm). The point with which the left reference mark coincides in this case will be point 3; after the floating mark is matched with it, the readings on the P and Q scales can be taken. In an analogous fashion, the floating mark is made to coincide with points 4, 5, and 6 of the model, for which definite positions are also set. Thus, to match the floating mark with point 4, the reading on scale x is set

equal to the reading for the second point, and on scale y , a reading equal to the reading for the third point. For point 5, the x -scale reading is equal to the reading for the first point, while the y -scale reading is equal to the reading for the first point, reduced by a constant quantity of 60 or 70 mm. For point 6, the x -scale reading is equal to the reading for the second point, while the y -scale reading is equal to the reading for the fifth point. Taking the reading Q on the scale of the transverse parallax screw for the first (or second) point as the initial reading, this value is subtracted from all the remaining readings. This will make it possible to obtain the quantities q for all remaining points and to calculate the elements of relative orientation. In the calculation, the quantity b is taken as the difference in the x -scale readings while the floating mark is matched with the second and first points, whereas the quantity y is a constant by which the readings are changed when the floating mark is successively matched with points 3, 4, 5, and 6.

The arrangement of the record for calculating the elements of relative orientation has the form given in the following Table.

Record for Determining the Elements of Relative Orientation
According to Measured Transverse Parallax

Pair No 1354-1356; $f_k = 68$ mm; $b = 80$ mm; $y = 70$ mm

Point No.	Transverse Parallax Scale Readings mm	q mm		Q mm	
2	+ 15.42	0			
4	+ 14.76	- 0.64			$\epsilon = - \frac{f_k + 343P}{2bv} \quad G_1 = + 25'$
6	+ 15.08	+ 0.56	$q_4 - q_6 = G_1$	- 0.20	$\epsilon = - \frac{f_k + 343P}{2bv} \quad G_2 = + 16'$
1	+ 15.42	0	$q_1 - q_5 = G_2$	- 0.25	
3	+ 14.92	- 0.50	$q_4 + q_6 = G_3$	- 0.08	$\epsilon = + \frac{f_k + 343P}{2v^2} \quad \frac{G_3 + G_1'}{2} = - 3'$
5	+ 15.77	+ 0.35	$q_3 + q_5 = G_3'$	- 0.15	

105. Relative Orientation on Duplicate Projectors

In contrast to the described method, the problem of determining the elements of relative orientation is solved in another manner, using duplicate projectors. In this case, in place of analytical determination of the elements of relative orientation, direct relative orientation of the two photos is performed. This task consists in the elimination of transverse parallax at six selected points by varying the spatial position of the photographs, which causes the projecting rays emanating from corresponding points to intersect.

In using this method, adjacent photographs are placed in the projection cameras of the instrument and are illuminated by a light source. Examining the images projected on the screen, the observer, by successive elimination of transverse parallax at six selected points, effects the proper relative orientation of the photographs.

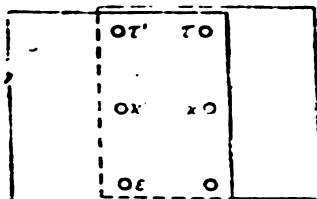


Fig. 186 - Sequence of Relative Orientation

To accomplish rapid orientation, the transverse parallax elimination is carried out according to a set pattern (Fig. 186). First, the transverse parallax is eliminated near the principal point of the left photograph by rotating the right photograph in its plane through the angle χ' , and then near the principal point of the right photograph by rotating the left photo-

graph through the angle χ . At the third point, the transverse parallax is eliminated by tilting the right photo at an angle of τ' , at the fifth point by rotating the right photograph about the base line through the angle τ , at the fourth point by tilting the left photograph at an angle of τ .

Since the transverse parallax at this point is influenced not only by the elements of relative orientation used for its elimination, the problem is solved by the

method of successive approximation. After eliminating the transverse parallax at one of the points, the observer reverts to all preceding points, eliminating the new transverse parallaxes at these points. This continues until the transverse parallax simultaneously disappears at the observed point and at all preceding points, which makes it possible to pass to the observation of the next point. To speed up the process of orientation, instead of completely eliminating the transverse parallax at point 5, it is increased but with reversed sign, in proportion to the ratio $\frac{f_k^2 + y^2}{2y^2}$, after which the observer again returns to points 1 and 2.

Thus, relative orientation of the photographs consists in giving them the relative position they occupied at the instant of exposure. This relative position is determined by five elements of relative orientation. Incorrect adjustment even of one of these leads to the appearance of transverse parallax, varying at different points of the model, and indicates that the corresponding projecting rays do not intersect and that there is no ground model formed. The photographs are oriented relative to each other without any geodetic control network, by eliminating the transverse parallax at five points of a stereo pair according to a set pattern, by the method of subsequent approximation. The result of this relative orientation will be a ground model, arbitrarily located relative to the screen of the duplicate projector.

106. Exterior Orientation of the Model

The ground model constructed is located (as shown) arbitrarily with reference to a geodetic system of coordinates, with whose XY plane the plane of the screen is considered to coincide. The quantities determining the position of the model relative to this system of coordinates are called elements of exterior orientation of the model. The elements of exterior orientation of the model consist of the three space coordinates X_S, Y_S, Z_S (Fig. 187); the left center of projection; the scale ratio $\frac{1}{m}$ of the model; the angle A formed by the vertical base plane and XZ plane

of the system of space coordinates; the angle μ between the principal base plane of the left photograph and the vertical base plane; and the angle of tilt γ' of the inclination of the base line relative to the horizontal ZY plane, measured in the principal base plane of the left photograph. These seven elements of exterior

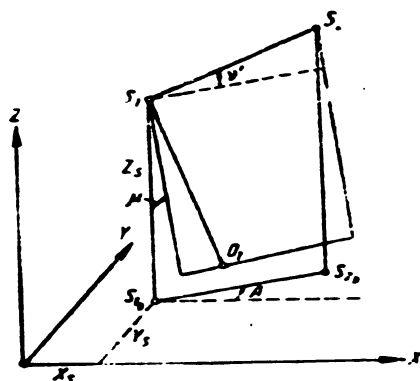


Fig. 187 - Elements of Exterior Orientation of the Model

orientation of the model together with the five elements of relative orientation, comprise the twelve quantities that determine the elements of exterior orientation of the two photographs.

To determine the elements of exterior orientation of the model, it is first necessary to find the non-collinear geodetic coordinates of three points on the surface of the model; for two of the points all three coordinates must be known, while for the third point it is sufficient to have only the elevation.

The coordinates for the points are measured in the system of coordinates of the model which permits finding the elements of exterior orientation of the model.

Actually, the mathematic relationship between the geodetic coordinates of the three selected points and their photogrammetric coordinates is expressed by the following equations*:

$$\begin{aligned} X_g - X_{S_1} &= m[(X_{ph} \cos \gamma' - Z_{ph} \sin \gamma') \cos A + (Y_{ph} \cos \mu - (Z_{ph} \cos \gamma' - X_{ph} \sin \gamma') \sin \mu) \sin A] \\ Y_g - Y_{S_1} &= m[(Y_{ph} \cos \mu - (Z_{ph} \cos \gamma' - X_{ph} \sin \gamma') \sin \mu) \cos A + (X_{ph} \cos \gamma' - Z_{ph} \sin \gamma') \sin A] \\ Z_g - Z_{S_1} &= m[(Z_{ph} \cos \gamma' - X_{ph} \sin \gamma') \cos \mu + Y_{ph} \sin \mu] \end{aligned} \quad (109)$$

* See Section 130 for derivation of these equations.

where X_g , Y_g , and Z_g are the geodetic coordinates of a selected point: X_{ph} , Y_{ph} , Z_{ph} are the photogrammetric coordinates of the same point. Equations (109) show that the unknowns are the seven elements of exterior orientation of the model, while all three geodetic coordinates X , Y , and Z of one point make it possible to derive three equations. Hence, to determine the unknowns, seven equations must be derived, which is possible if all three coordinates of two of the points and the elevation of the third point are known. To check the measurements and calculations, all three geodetic coordinates of four points of the model are usually determined. In operating with a duplicate projector, the elements of exterior orientation of the model generally are not determined; rather the exterior orientation is performed by optico-mechanical or graphic methods.

In the optico-mechanical solution of the problem the observer, as shown in Section 100, uses a tracing table to locate the position of two model points on the screen, which all three geodetic coordinates are known and, by varying the elevation of the screen, determines their relative vertical intervals as well as the relative vertical interval of the third point. Then, by measuring the distance l between two points on the screen and using the equation

$$d = \sqrt{l^2 + \Delta h^2} \quad (110)$$

it is possible to calculate the distance between two points on the surface of the model, if Δh is the relative elevation of one point over another, as measured with the tracing table. The expression

$$D = \sqrt{(X_{g1} - X_{g2})^2 + (Y_{g1} - Y_{g2})^2 + (Z_{g1} - Z_{g2})^2} \quad (111)$$

is used to determine the distance D between the same two points according to their geodetic coordinates, so that the scale of the model can be obtained from

$$\frac{d}{D} = \frac{1}{M} \quad (112)$$

The geodetic elevations of all three points are then multiplied by the scale ratio $\frac{1}{m}$, expressed in the scale of the model. Assuming that the geodetic elevation, at the scale of the model, of one of the points is equal to the measured elevation obtained on the tracing stand, it is possible to obtain the geodetic elevation of the two other points relative to this initial point. Then, if the geodetic elevations are set on the scales of three stands which are then placed over the points on the screen, at correct exterior orientation of the model, the marks on the stands will coincide with the corresponding points of the model. At an arbitrary position of the model relative to the plane of the screen, the marks of the three stands will not coincide with the corresponding points of the model; to make them coincide, the two projection cameras are rotated together, relative to the screen, about two mutually perpendicular axes. Instead of turning the two cameras relative to the fixed screen, it is possible to match the marks of the three stands with the points of the model, by rotating the screen with respect to the fixed projection cameras. After matching the marks of the three stands with the corresponding points of the model, the resultant model is considered correctly oriented relative to the screen plane, which makes it possible (by a method described in Section 100), to construct the ground plan on the scale of the model. For conversion to the required scale, the map constructed on the duplicate projector is reduced photographically or plotted by a pantograph. Naturally, in plotting relief, the readings on the scales of the tracing stand are converted to vertical intervals, using the proper scale factor.

The exterior orientation of the model by the graphic method is also carried out on the basis of geodetic coordinates of three points of the model. To accomplish this, the photogrammetric coordinates of all points required for preparing the map are measured on the model, including the three control points. The position of all selected points is noted on a sheet of paper placed on the screen of the instrument, and their height in millimeters is read on the scale of the trac-

ing stand. By comparing, as explained above, the distance between the points of the model with the corresponding ground distance, the scale ratio of the model is determined; the denominator is then used to multiply all photogrammetric elevations. Taking one of the control points as the datum point, its elevation mark (both geodetic and photogrammetric) is subtracted from the marks of two other points. Then, if the

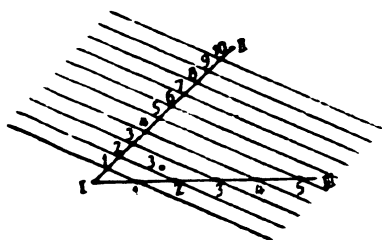


Fig.188 - Graphic Determination of Corrections in Photogrammetric Elevations

photogrammetric and geodetic systems of coordinates are parallel, the marks for the control points will be equal in both systems. If they are not equal, one of the systems of coordinates must be tilted relative to the other system.

Since it is possible on the basis of eq.(109) to write, with a sufficient degree of accuracy:

$$Z_g - Z_{ph} = -X_{ph} \sin \nu' + Y_{ph} \sin \nu \quad (113)$$

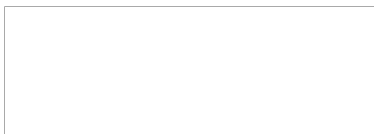
The difference in geodetic and photogrammetric marks will be directly proportional to the angles of tilt of the model and to the current coordinates of the points of the model. Therefore, knowing these differences for two control points (at the datum point the difference is equal to zero), it is possible to find the corresponding differences for all remaining points of the model by linear interpolation of the distances between the control points. To this end, straight lines connecting the control points are drawn on the sheet of paper on which the plan position of the points of the model is entered, and these are linearly interpolated proportional to the differences of the geodetic and photogrammetric marks.

Let the control point I (Fig.188) be taken as the datum point for the geodetic and photogrammetric marks, and let the divergence between geodetic and photogrammetric marks at points II and III be +10.7 m and +5.4 m, respectively. By dividing the distance I-II into 10.7 parts, we obtain on the straight line I-II the position

of the points where the divergence between geodetic and photogrammetric marks is 1, 2, 3...m. In the same manner, the distance I-III is divided into 5.4 parts and each marked point will indicate a discrepancy in multiples of one meter. If the points marked on I-II and I-III and having identical divergences are joined by straight lines, the entire sheet of paper will be divided into zones corresponding to divergences for any points on the model. Thus, e.g., for the point 3, the divergence read on the graph is equal to +2.3 m, so that its geodetic mark will be equal to the photogrammetric mark plus the read-off correction. This construction makes it possible to determine the marks of all points of the model in the geodetic system of coordinates.

Thus, the exterior orientation of the model consists in a determination of seven unknown quantities which can be found on the basis of the geodetic coordinates of three points of the model, two of which have all three coordinates given while the third has only the elevation. All three geodetic coordinates of two points are needed to determine the scale of the model, which is found as the relationship of segments on the model surface and on the ground.

Instead of determining the elements of exterior orientation of the model, exterior orientation is accomplished by tilting both projection cameras (or the screen) until the control points coincide with corresponding points of the model. The spatial position of the control points is fixed by the tracing stands adjusted in accordance with the geodetic elevation mark of the scale of the model. After exterior orientation of the model, it is measured and the ground plan is constructed. When densifying the topographic elevation control network by means of duplicate projectors, the exterior orientation of the model is often replaced by graphic application of corrections to the reference marks of points of the model, obtained by linear interpolation of the differences in geodetic marks and reference marks of the model points for three control points.



107. The Undistorted Model Method

Photogrammetric densification of the geodetic elevation net can also be accomplished by the undistorted model method, proposed by G.V. Romanovskiy and M.D. Konshin. This method permits the determination of elevation reference marks of photographed points by linear interpolation of the errors in vertical intervals obtained for three points with geodetic elevation marks.

As shown in Section 76, a correction for the horizontal parallax difference at changes in the elements of exterior orientation is expressed by the relation

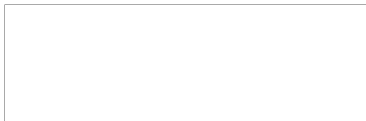
$$\begin{aligned} \delta p = \frac{x}{f_k} \left(\delta H + \frac{2b}{\rho} \alpha_{x_2} \right) + \frac{x^2}{f_k \rho} (\alpha_{x_1} - \alpha_{x_2}) + \frac{xy}{f_k \rho} (\omega_1 - \omega_2) + \\ + \frac{y}{\rho} \left(\chi_1 - \chi_2 + \frac{b}{f_k} \omega_2 \right) \end{aligned} \quad (50)$$

An error in the horizontal parallax difference leads to errors in the vertical intervals being determined; these errors will have a non-linear relation with the current coordinates of the observed points. This fact makes it impossible to perform a linear interpolation of the variation in horizontal parallax difference or vertical interval errors. The nonlinear character of the errors is due to the effect of the second and third terms of eq. (50), so that linear interpolation becomes possible if the effect of these terms is eliminated.

The second and third terms in eq. (50) are the result of relative longitudinal and relative lateral angles of tilt of the photographs which affect not only the variation in horizontal parallax difference, but also the magnitude of traverse parallax, expressed by the equation

$$q = -\frac{xy}{f_k \rho} + \frac{x'y'}{f_k \rho} + \frac{f_k^2 + y^2}{f_k \rho} \epsilon + \frac{x}{\rho} \chi - \frac{x'}{\rho} \chi' \quad (100)$$

On the basis of Fig. 139, it is possible to establish the following relation be-



tween the angles τ , τ' , $\alpha_{x_1} - \alpha_{x_2}$ and ν , where ν is the angle of tilt of the base

$$\begin{aligned} \gamma &= 180^\circ - \varphi - (180^\circ - \varphi') = \varphi' - \varphi = \alpha_{x_1} - \alpha_{x_2} = \\ &= \tau - 90^\circ - \tau' + 90^\circ = \tau - \tau'; \\ \varphi' &= \nu + 90^\circ - \alpha_{x_2} = -\tau' + 90^\circ \end{aligned}$$

Hence, noting that $\sin \nu = + \frac{H_2 - H_1}{B} = - \frac{\Delta H}{B}$ it is possible to write:

$$\tau' = + \frac{\Delta H}{B} \rho + \alpha_{x_2} - \alpha_{x_1} - \nu$$

When the photographs are accurately oriented along the initial radial, the transverse parallax at the first and second points is equal to zero; hence, for these points ($x_1 = y_1 = 0$; $x_2 = +b$; $y_2 = 0$)

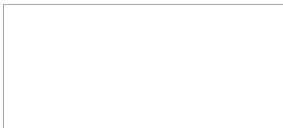
$$+ \frac{f_k}{\rho} \varepsilon + \frac{b}{\rho} \chi' = 0; + \frac{f_k}{\rho} \varepsilon + \frac{b}{\rho} \chi = 0, \text{ or } \chi = \chi' = - \frac{f_k}{b} \varepsilon$$

In addition, on the basis of Fig.190, $\varepsilon = + (\omega_2 - \omega_1)$.

Substituting all expressions obtained in eq.(100) and assuming that $x' = x - b$, we have:

$$\begin{aligned} q &= - \frac{xy}{f_k \rho} \tau + \frac{xy}{f_k \rho} \tau' - \frac{by}{f_k \rho} \tau' + \frac{f_k}{\rho} \varepsilon + \frac{y^2}{f_k \rho} \varepsilon + \\ &+ \frac{x}{\rho} \chi - \frac{x}{\rho} \chi' + \frac{b}{\rho} \chi' = - \frac{xy}{f_k \rho} (\alpha_{x_1} - \alpha_{x_2}) - \\ &- \frac{by \Delta H}{f_k B} - \frac{by}{f_k \rho} \alpha_{x_2} - \frac{y^2}{f_k \rho} (\omega_1 - \omega_2) \end{aligned} \quad (114)$$

An analysis of eqs.(50) and (114) indicates that they have much in common due to the coincidence of several terms when the transverse parallax is multiplied by



the ratio $x : y$. In this case,

$$\begin{aligned} \frac{qx}{y} = & -\frac{x^2}{f_k \rho} (\alpha_{x_1} - \alpha_{x_2}) - \frac{x}{f_k} \delta H - \frac{bx}{f_k \rho} \alpha_{x_2} - \\ & - \frac{xy}{f_k \rho} (\omega_1 - \omega_2) \end{aligned} \quad (115)$$

A summation of eqs.(50) and (115) yields

$$-\delta p + \frac{qx}{y} = \frac{bx}{f_k \rho} \alpha_{x_2} + \frac{y}{\rho} (x_1 - x_2 + \frac{b}{f_k} \omega_2) \quad (116)$$

which corresponds to a change in the expression $(-\delta p + \frac{qx}{y})$, directly proportional to the change in the coordinates of the current points. These theoretical considerations form the basis for the undistorted model method since they lead to an expression, connected by a linear relation with the coordinates of the current points.

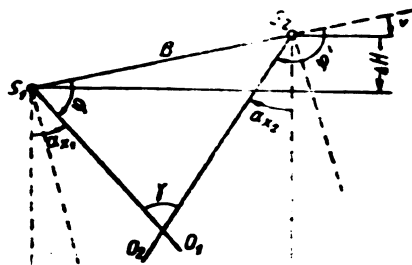


Fig. 180 - Relation of Longitudinal Angles of Tilt and Difference in Flight Altitude

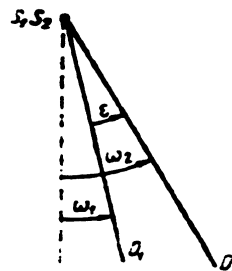


Fig. 190 - Relation of Longitudinal Angles of Tilt

For this reason, if, at certain points on a stereo pair the horizontal parallax difference $\Delta'p$ and the transverse parallax are measured and if the latter, multiplied by the coefficient $x : y$ in the case of transverse parallax, is subtracted

from the former, then the resultant expression will have only linear distortions, i.e.,

$$\Delta' p - \frac{qx}{y} = \Delta p + \delta p - \frac{qx}{y} = \Delta p - \frac{bx}{f_k \rho} \alpha_{x_2} - \frac{y}{\rho} \left(x_1 - x_2 + \frac{b}{f_k} \omega_2 \right) = \Delta'' p \quad (117)$$

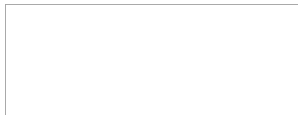
The elevation of points calculated according to $\Delta'' p$

$$h'' = \frac{H \Delta'' p}{b + \Delta'' p} = \frac{H \Delta'' p}{b} = \frac{H}{b} \Delta p - \frac{H}{b} \left[\frac{bx}{f_k \rho} \alpha_{x_2} + \frac{y}{\rho} \left(x_1 - x_2 + \frac{b}{f_k} \omega_2 \right) \right] = h + \delta h \quad (118)$$

will contain an error proportional to the current coordinates, which error can be interpolated linearly. In this case, knowing the difference δh between the geodetic and photogrammetric elevation reference marks of three points, the intermediate points can be interpolated proportionally to the distance between the points and a graph of δh corrections for all the remaining points of the model can be constructed. Subtracting these corrections from the photogrammetric elevations of the points of the model, the elevations for these points can easily be obtained in the geodetic system of coordinates.

The solution of this problem is simplified by the fact that the ratio of the transverse parallax to the ordinates also has a linear relation with the current coordinates of the model points; accordingly, knowing the ratio $q:v$ for three points of the model, it is possible to obtain the corresponding ratio for all remaining points, again by linear interpolation.

The practical solution of this problem is as follows: Let (Fig.191) three points I, II, and III located within the limits of a stereo pair have geodetic elevation marks; wanted are the elevations for points 1, 2, 3...k of the model. For



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Calculation of Elevation by the Undistorted Model Method

Pair No. _____

H = 2100 m; b = 70 mm

Point No.	x mm	x - x ₀ mm	p mm	Δp mm	y mm	y - y ₀ mm	Q	q mm	$\frac{q}{y}$	$\frac{q}{y} x$ mm	Δ ^{II} p mm	h ^{II} m	Δh m	h m	A m
I	154.3	0	68.34	0	173.4	0	14.35	0							
I	236.8	+ 82.5	71.25	+ 2.91					-0.01260	-1.04	+ 3.95	+ 110.8	+ 110.8	0	478.2
												+ 3.6	+ 98.1	- 66.5	411.7
II	158.1	+ 3.8	69.38	+ 1.04					-0.00860	-0.03	+ 1.07	- 87.8	+ 119.9	207.7	270.5
												+ 27.7	+ 101.8	159.5	348.7
III	190.4	+ 36.1	65.17	- 3.17					-0.00990	-0.36	- 2.81	+ 71.0	+ 102.2	- 31.2	447.0
I	173.4	+ 19.1	67.24	- 1.10					-0.00930	-0.19	- 0.91				
2	185.7	+ 31.4	70.48	+ 2.14					-0.01000	-0.31	+ 2.45				
.											
.											
.											
.											
k	199.5	+ 45.2	69.15	+0.81					-0.01045	-0.47	+ 1.28	+ 37.1	+ 117.5	- 80.4	397.8
b					256.8	+ 83.4	13.54	- 0.81	-0.00974						
a					90.2	- 83.2	15.06	+ 0.71	-0.00853						
c					259.5	+ 86.1	13.30	- 1.05	-0.01219						

this three points, a, b, and c are selected, at which the ordinates and transverse parallaxes are measured. These points are so selected that they are located close to the corners of the stereo pair and coincide with well-marked contours, thus ensuring

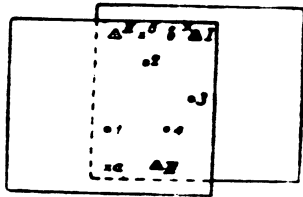


Fig. 191 - Selection of Points in the Undistorted Model Method

accurate measurement of transverse parallax. The photograph is placed on the carrier of a stereocomparator and oriented according to the initial radial. After this, at points I, II, III, 1, 2, 3, ..., k the horizontal parallax differences and the abscissas are measured relative to the principal point of the left photograph. The data obtained are recorded in Columns 2, 3, 4, and 5 of the record (page 371); in Column 2, the x-scale reading of the stereocomparator is entered and, in Column 4, the reading from the scale of the horizontal parallax screw.

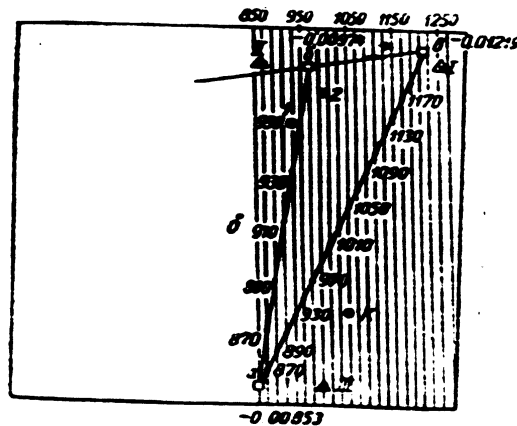


Fig. 192 - Graphic Interpolation of the Ratio $q : y$

during the reduction to the principal point of the left photograph, are subtracted from all remaining readings, and the differences are entered in Columns 3 and 5. At

the points a, b, and c of the model, the transverse parallaxes and ordinates are measured relative to the principal point of the left photograph and the results are entered in Columns 6 and 8, while the differences in readings for all points and the reading at the principal point are entered in Columns 7 and 9. Column 10 is to give the ratio of the transverse parallax to the ordinate, plotted on the tracing (Fig. 192), which is a copy of the location of all points on the left photograph. The

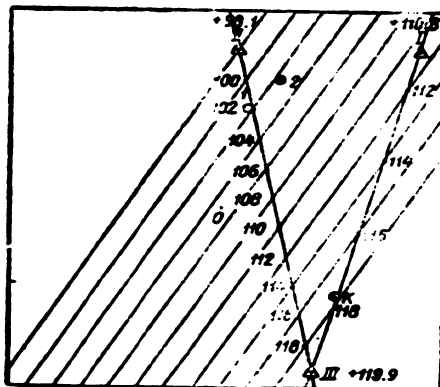


Fig. 193 - Graphic Determination of Corrections for Photogrammetric Elevations

points a, b, and c are connected by straight lines, and the radials ab and ac are interpolated proportionally to the differences of the ratio $q:y$ for the terminal points of these radials, noting on the plotting paper the points at which the ratio is equal to -0.00853 , -0.00974 , etc. Connecting the points of the radials ab and ac with points of the same ratio by straight lines, yields an interpolation graph which permits finding the $q:y$ ratio for all remaining points of the model.

These ratios are then entered in Column 10.

In Column 11, the product of the resultant ratio and the abscissa is entered, and this is subtracted from the measured horizontal parallax difference Δp to give the data ($\Delta''p$) to be entered in Column 12. From the magnitude of $\Delta''p$ the elevations of the model points are calculated and entered in Column 13. For three points of the model having geodetic elevations, these data are entered in Column 16 and, taking one of these points as the datum, their geodetic elevation differences are entered in Column 15. The difference between the geodetic and photogrammetric elevation difference is entered in Column 14, and these data are used to construct a second interpolation graph of the elevation errors (Fig. 193), analogous to the first graph.

From the interpolation, the difference between the geodetic and photogrammetric elevation of all remaining points (Column 14) are obtained, which are subtracted from the data in Column 13 to give the elevations in the geodetic system of coordinates (Column 15). By adding these elevation differences to the elevation of the datum point, will give the elevation of all points of the model in the geodetic system of coordinates.

The method described for determining the elevations of points on the model is modified in the case of prominent ground relief. This fact, however, does not limit the usefulness of the undistorted model method. The arbitrary arrangement of points with known geodetic elevations, simplicity of operations, and a high degree of accuracy are the basic advantages of this method.

108. Schematic Diagram of a Stereoplanigraph

Duplicate projectors include the stereoplanigraph which is used in the compilation of large-scale maps. This instrument differs from the above-described types of duplicate projectors. Like all the duplicate projectors, the stereoplanigraph (Fig. 194) is equipped with two projection cameras whose elements of interior orientation are identical to those of the mapping camera. Instead of projecting both images onto a common screen with a light spot, the stereoplanigraph uses two screens with two marks which must be geometrically superposable. Thus, the spatial triangle formed by the two projecting rays and the base line becomes a trapezoid formed by the distance between the nodal points of the lenses of the two cameras, the projecting rays, and the distance between the two marks. Hence, the projection base in the stereoplanigraph is the difference of the distance between the nodal points of the camera and the distance between the marks.

The existence of two marks in the stereoplanigraph makes the instrument useful for setting up the various magnitudes of the projection base. For example, if the distance between projection centers remains constant, while the distance between the

marks is increased, this will correspond to a shortening of the projection base which, for this reason, can be made equal to zero (if the distance between the marks equals the distance between the centers of projection), or can even assume a negative value. By analogy, if the right mark is raised, the right projection center

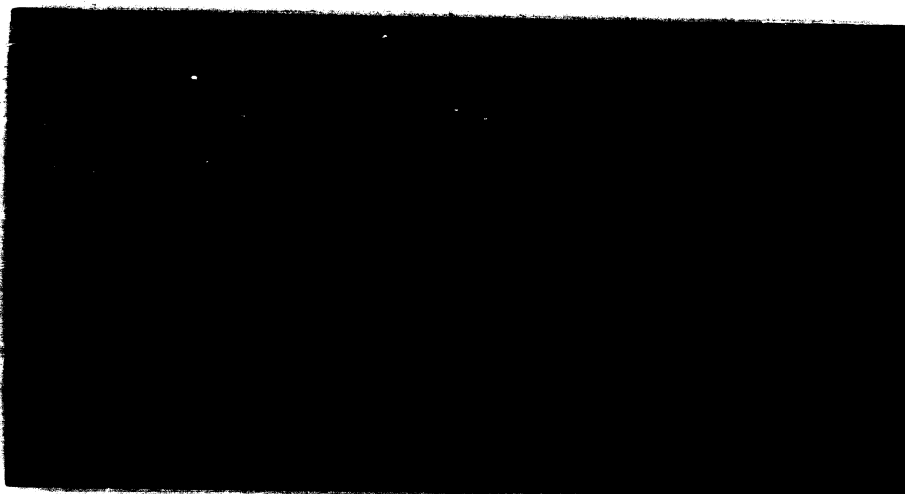


Fig. 194 - The Stereoplaniograph

will be below the left center in exactly the same way as at a forward displacement of the right mark, the left center of projection is displaced backward. Thus, the elements of exterior orientation defined by the projections B_x , B_y , and B_z of the projection base on the axes of coordinates may be set on the instrument by moving the marks instead of the projection cameras.

The angular elements of exterior orientation of the cameras in a stereoplanigraph are set in the same way as the duplicate projector by tilting and rotating each camera about three axes. The two marks are matched with corresponding points of the image of the two photographs by simultaneous shifting of the marks along the XX axis of the instrument, with the projection cameras remaining stationary and by simultaneous shifting of both cameras along the YY and ZZ axes of the instrument, with the two dots remaining fixed. For such displacements, the instrument is equipped with solid coordinate axis spindles, on which the carriers with the marks and projecting cameras are moved.

In view of the fact that the images of the corresponding points on the two photographs are projected on two different marks, the stereoplanigraph observation is accomplished by the usual stereoscopic method as in stereoscopes. However, the adjustable position of the marks and projection cameras makes it necessary to resort to a rather complex optical system.

Operation of the instrument is according to the earlier described method. Negatives are placed in the cameras, illuminated by the light source, and projected through the lens onto the plane of the marks. During stereoscopic viewing, the observer will see a ground model and a single floating mark. Since the initial position of the projection cameras was rather arbitrary, the corresponding points will not simultaneously coincide with both marks, and the existence of transverse parallax indicates that the position of the two carriers is incorrect.

Elimination of transverse parallax at six points of the model makes it possible to obtain correct relative orientation of the two photographs. By comparing the coordinates of three points on the model with their geodetic coordinates permits exterior orientation of the model. After measuring the coordinates of various parts of the model, a map of the photographed terrain can be prepared.

In constructing the map, displacement of the carriers of the instrument along its axes of coordinates causes displacement of the pencil of the coordinate graph to

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trace a map of horizontal contour lines on the plotting board, which is placed on a special table. For convenience, the scale of the plan to be drawn can be varied within sufficiently wide limits, making it possible to construct this plan to the specified scale. After processing the first stereo pair, the second photograph may be left in the projection camera and the first photograph replaced by a third, after which the previous processing is repeated. However, in this case, the projection base must have a negative value and the viewing system for viewing the stereo model with a direct stereo effect should have the line of sight so set that the second photograph is viewed with the same eye that had viewed the first, while the third photograph is viewed with the eye that had viewed the second.

109. The Principle of Spatial Phototriangulation

For exterior orientation of the constructed model, the geodetic coordinates of three points must be known within the limits of each model; these give basic values for much of geodetic field operations. The shortening of this base is possible if the photogrammetric method of spatial phototriangulation is used. This method is designed for densification of the basic elevation control network.

Let (Fig.195) $S_1S_2S_3S_4$ be the centers of projection of four adjacent photographs having an end lap of 60%, from which the points 1,2,3 of the terrain are represented simultaneously on the first three photographs, and 4,5, and 6 are depicted on the second, third, and fourth photographs. If the first two photographs are placed in the projection cameras of the duplicate projector and mutually oriented, then the coordinates of points 1,2, and 3 of the first model will be obtained, relative to its system of coordinates. the position of the two centers of projection (S_1 and S_2) are determined in this system of coordinates. In this way, after relative orientation of the second and third photographs, the position of points S_2 , 1, 2, and 3 can be determined, but this time in the system of coordinates of the second model. If the distance $S_2 - 1$ in the second model is divided by the corresponding

distance $S_2 - 1$ measured on the first model, the resultant ratio will indicate the scale of the second model relative to the first, permitting a conversion of the coordinates of all points of the second model into the scale of the first model.

Then, assigning values obtained from the first model to the coordinates of the

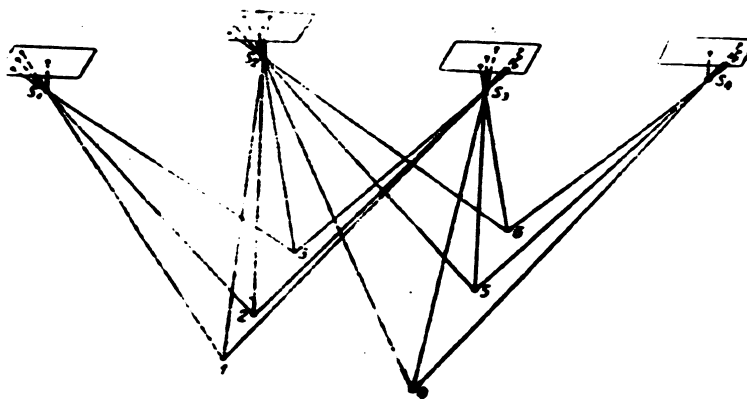


Fig. 195 - Schematic Diagram of a Spatial Phototriangulation Grid

points S_2 of the second model and changing the coordinates of all remaining points to the second model by the same quantity, a common origin of coordinates for both models is obtained. The difference in elevation reference marks of points 1,2,3 of the first and second model indicates their different angular orientation. Hence, by rotating the second model relative to the three axes of the space coordinate, it is possible to bring about equality of the coordinates of points 1,2,3 of the second model with the first model. As a result of these operations, the second model will have the same spatial orientation and the same scale as the first. Subsequently, a third model consisting of the third and fourth photographs and having the points S_3 , 4,5, and 6 in common with the second model may be connected in the same manner.

Continuing as before, it is possible to obtain a single spatial model of the entire flight strip, consisting of a series of models having the same position in space. For this reason, knowing the geodetic coordinates of three non-collinear points lying along this strip, its exterior orientation can be carried out in accordance with the method given in Section 106. In this case, the extent of the basic geodetic grid is determined by the length of the flight strip which makes up a single spatial phototriangulation net.

The development of spatial phototriangulation series is carried out by analytical or optical mechanical methods. Of the analytical methods, the differential method of spatial phototriangulation developed by the Central Scientific Research Institute of Geodesy, Aerial Photography, and Cartography is much used at present.

In this method, the elements of relative orientation of all adjacent photographs are determined by measuring the transverse parallax on the stereocomparator, followed by analytical computations from eqs. (104) - (108). Using as the original angles of tilt of the first photograph those determined from relative orientation of the first stereo pair, the relative longitudinal and lateral angles are added to them, obtaining thus the angles of tilt for all photos, relative to the initial. This calculation is done according to the equations:

$$\left. \begin{aligned} \alpha_{x_i} &= \alpha_{x_0} + \sum \Delta \alpha_x \\ \omega_i &= \omega_0 + \sum \epsilon \end{aligned} \right\} \quad (119)$$

where α_{x_i} and ω_i are the longitudinal and lateral angles of tilt of any photograph; α_{x_0} and ω_0 are the longitudinal and lateral angles of tilt of the original first photograph; and $\Delta \alpha_x$ and ϵ are the relative longitudinal and lateral angles of tilt. Likewise, the equation

$$\Delta H_i = Bv \quad (120)$$

yields the vertical interval of any center of projection relative to the preceding

one, where v is the inclination of the base line.

The elements of exterior orientation of all photographs relative to the original are called arbitrary or conditional elements of exterior orientation since they determine the position of all photographs relative to a single plane, accepted conditionally as horizontal. Presence of the conditional elements of exterior orientation permits determining the position of conditional nadir points according to

$$\left. \begin{aligned} \Delta x &= f_k \tan \alpha_{x_i} \\ \Delta y &= f_k \tan \omega_i \end{aligned} \right\} \quad (121)$$

where Δx and Δy are the distance of the conditional nadir point (foot of the perpendicular dropped from the center of projection to a plane conditionally accepted as horizontal) from the principal point of the photograph. If the conditional nadir point is taken as the apex of radials in the plane of the photograph, these radials will contain no errors caused by ground relief. For this reason, in carrying out the densification of a horizontal phototriangulation control grid in mountainous regions, the radial center is always taken as the same arbitrary or conditional nadir point for which, in such region, the elements of relative orientation of the photographs must be taken prior to such densification.

After determining, in this way, the elements of exterior orientation of all photographs with respect to a single common datum plane, and measuring the horizontal parallaxes of points located in the zone of triple overlap, the corrections for them may be calculated by the following formula:

$$\Delta p = \frac{x_1}{f_k} H + \frac{x_1^2}{f_k^2} x_{10} - \frac{x_2^2}{f_k^2} x_{20} + \frac{x_1 y_1}{f_k^2} x_{10} - \frac{x_2 y_2}{f_k^2} x_{20} \quad (122)$$

to obtain the horizontal parallaxes corrected with respect to the same datum plane. By calculating the elevation differences from the corrected horizontal parallaxes, the photogrammetric elevation readings are found for all points with respect to the

datum plane; comparing three of these photogrammetric readings with the geodetic data and using graphic exterior orientation, it is possible to convert to the geodetic elevation readings.

When the statoscope is read during a flight, this procedure is somewhat modified, owing to the fact that the statoscope makes it possible to find the difference ΔH of the flight altitudes. Therefore, the angle of tilt v of the base line is found from the relation

$$v = \frac{\Delta H_i \rho}{B} \quad (123)$$

which permits us to find the longitudinal angle of tilt of the photograph from

$$\alpha_{x_i} = v_{i-1} + \tau'_{i-1} = v_i + \tau_i \quad (124)$$

The lateral angle of tilt of the photograph is determined in the same way as in the previous case.

This solution of the problem permits the longitudinal angles of tilt of the photographs to be found independently of one another, in contrast to the method used when no statoscope readings are available. The independent determination of these angles assures a considerably slower accumulation of errors in the spatial phototriangulation grid when the number of models constituting the model of the flight strip is large.

If the altitude readings of the principal points of all photographs are known in advance, the degree of graphical exterior orientation is carried out separately for each component of the model. In this case there will be no accumulation of errors at all in the spatial phototriangulation grid along the flight strip.

110. The Multiplex

For constructing a special phototriangulation grid by an optico-mechanical method, a special instrument known as the multiplex has been designed. It consists

of a series of projecting cameras operating on the same principle as the duplicate projector. The multiplex (Fig.196) consists of the base 1, carrying two vertical standards 2. Turning the hand wheel 3 will rotate the carrier bar 4, which represents the XX axis of the instrument, about the uprights 2. On the carrier bar 4 six to eight supporting columns 5, bearing the projecting cameras 6, are mounted. Each support 5 consists of two cross-tracks, perpendicular to the bar 4; spindles about

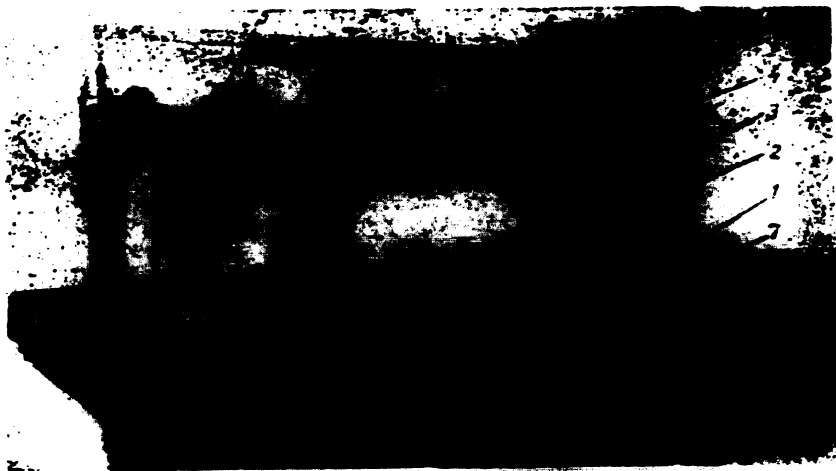


Fig. 196 - The Multiplex

which the projecting cameras can be rotated are attached to the movable portion of the support. The cross-tracks of the support represent the YY and ZZ axes of the instrument; motion along these tracks, together with a displacement of the support along the carrier bar 4, allows each center of projection to be properly positioned with respect to the selected system of space coordinates. Angular orientation of the projecting cameras is obtained by rotating them about spindles through the angles α_x and ω as well as about the principal ray through the angle χ . The projection camera 6 contains a projecting lens, a plate holder carrying the photograph, and a light source consisting of an electric bulb and a condenser directing a beam of

light rays into the projecting lens.

To reduce the size of the instrument and the scale of the model constructed from the original negatives, reduced diapositive plates are prepared from original negatives and a number of copies made. To preserve the respective position of the projecting rays, the preparation of the reduced copies must be accompanied by a proportional reduction of the focal length of the projecting camera. The focal lengths of the projecting cameras of a multiplex projector are, therefore, several times shorter than the focal length of the mapping camera.

The image of the reduced copies, placed in the projecting cameras, is projected

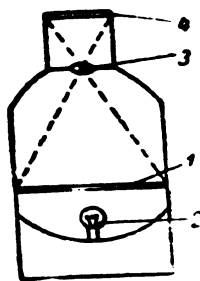


Fig. 197 - Schematic Diagram of a Reducing Printer

onto a single common screen. To increase the accuracy of observation, anaglyphic optical filters are used. For this purpose, a slot is provided in the illuminator of each projecting camera, into which the proper color filter is inserted, and the image is viewed through spectacles whose lenses serve as optical filters. The model so constructed is measured with the aid of the stand 8, equipped with a light spot described in Sect. 100.

The reduced copies are prepared from the negatives on a special reducing printer whose diagram is given in Fig. 159. The image of the negative 1, illuminated by the light source 2, is projected through the lens 3 onto the plane 4, on which a diapositive plate is placed. The distances from the negative to the lens and from the lens to the image, are adjusted according to the focal lengths of the projecting and mapping cameras. Thus, e.g., at a focal length of 25 mm for the projecting camera and 100 mm for the mapping camera, the scale ratio would equal 4, in accordance with which the proper distances are set on the reducer, i.e., the distance from the negative to the lens must be four times the distance from the lens to the dia-

positive plate. Obviously, at such a reduction, the focal length of the reducer lens must be computed according to the equation for optical conjugation:

$$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f_{ob}}$$

where d_1 is the distance from the negative to the lens, d_2 the distance from the lens to the diapositive plate, and f_{ob} the focal length of the lens of the reducing printer.

In the example given, $d_1 = 4d_2$ and, therefore,

$$\frac{1}{4d_2} + \frac{1}{d_2} = \frac{1}{f_{ob}}$$

or

$$d_2 = \frac{5}{4} f_{ob}$$

When the reduced copies are finished, they are placed in the projecting cameras of the multiplex, with their principal points matched with the corresponding fiducial marks on the plate holder or with a special centering lens, and the image is projected on the screen of the stand, located on the screen of the multiplex itself. Viewing the projected images, the observer eliminates the transvers parallax to obtain relative orientation of the photographs (Sect. 105) while the intersection of the corresponding projecting rays forms a model of the terrain. Selecting three points of this model, located in the zone of triple end lap, the observer matches the light spots of the three stands with these points.

Passing to the second pair, the light is turned off in the first projecting camera, and turned on in the third. The second and third photographs are then mutually oriented. However, since during the mutual orientation of the first two photographs, the second occupied the correct position with respect to the first, the position of the second photograph cannot now be changed when it is mutually oriented with the third. Therefore, the mutual orientation of the second and third photo-

graphs is effected by moving only the third.

In this case, the transverse parallax at six selected points is eliminated in sequence. The transverse parallax at the second point is eliminated by shifting the third projector along the Y axis (by a quantity b_y); at the first point, by rotating it through the angle $\Delta\chi$; at the third point, by a longitudinal tilt $\Delta\alpha_x$; at the fifth point, by a longitudinal tilt ϵ ; and at the fourth point, by shifting along the Z axis (by a quantity b_z). This system of eliminating parallax is based on the expression relating transverse parallax to the elements of relative orientation:

$$q = -\frac{xy}{f_k \rho} \Delta\alpha_x - \frac{y}{f_k} b_z + \frac{x}{\rho} \Delta\chi + b_y + \frac{f_k^2 + y^2}{f_k \rho} \epsilon \quad (125)$$

This equation is derived from eq.(100) if we consider

$$x' = x - b; \Delta\alpha_x = \tau - \tau'; \frac{b}{\rho} \tau' = b_z; \frac{b}{\rho} x' = b_y; \Delta\chi = \chi - \chi'$$

As a result of the relative orientation, the second pair is given the same spatial orientation as the first, in view of the fact that the direction from the second center of projection still remains unchanged. To make the scale of the second model equal to that of the first, the three points of the first model, marked by the stands, must be made to coincide with the corresponding points of the second model. This is done by moving the third projector in the direction of the base line until the three points of the model so selected coincide with the light spots of the stands. All subsequent photographs are oriented in the same way, as a result of which the model is given a single scale and a single spatial position.

The exterior orientation of the constructed model of the flight strip is accomplished with the aid of three stands arranged according to the geodetic coordinates of the control points, by the simultaneous tilting of all the projecting cameras about two mutually perpendicular axes (see Sect.106). This tilting is accomplished in the longitudinal direction by tipping the carrier bar and in the lateral direc-

tion, by rotating it. Thus, when work on the multiplex is confined merely to the densification of a geodetic control net, such exterior orientation of the model is not performed. In this case, after the photographs have been mutually oriented and reduced to the scale of the preceding model, the coordinates of all points of densification are measured on the surface of the model by the method described in Section 100. The coordinates so obtained for points selected on each separate model will be referred to one system of coordinates and represented on a single scale. The scale of the model of the flight strip may then be found and the coordinates of the selected points may be determined in the geodetic system of coordinates.

111. Preparation of a Graphic Plan on the Multiplex

Besides the photogrammetric densification of an existing geodetic control net by the method of spatial phototriangulation, the multiplex is also used for the compilation of the plan itself. In this case, after successive orientation of all photographs with respect to the datum, and reduction of the individual models to a common scale, the exterior orientation of the model of the flight strip is performed by the optical-mechanical method. In this way, all the photographs will be brought into correspondence with the elements of exterior orientation with respect to the plane of the screen. Since, under the conditions of operation, it is advisable to place the plane of the screen 250 - 300 mm from the projecting lenses, this distance also determines the approximate scale of the model. For instance, in processing photographs with a focal length of 70 mm, the scale of the model will be approximately four times the scale of the photograph, and with a focal distance of 100 mm it will be from 2.5 to 3 times that scale.

To compile the graphic plan, a drawing board must be placed on the multiplex screen, onto which a stand with a small screen is mounted. When the first two projecting cameras, with the anaglyphic optical filters, are illuminated the observer, viewing the images thrown on the small screen through anaglyphic spectacles, per-

1

ceives a spatial model of the photographed terrain. This model will be intersected by the plane of the small screen at points having the same elevation readings. By shifting the small screen vertically and the stand along the plane of the board, the light spot may be directly matched with various points of the photographed contour; the pencil of the stand will then trace its motion on the board. In exactly the same way, knowing the scale of the constructed model, we may pass from readings on the scale of the stand to elevations on the terrain and, consequently, after establishing definite readings on the scale corresponding to the selected sections, we may also draw the horizontals on the board.

After processing the first stereo pair, the light in the first projection camera is turned off and the third is turned on. Here, while leaving the previous light filter in the second projection camera, the position of the lenses in the spectacles must be changed, or the inverse stereo effect will be obtained. As a result of such processing a horizontal plan will be obtained for all photographed areas on the given pictures. It is usually reduced to the assigned scale of the plan to be prepared by photographic reproduction or by using a pantograph that can be attached to the multiplex.

112. Photopolygonometry

The densification of an existing elevation control network by a photogrammetric method can be done not only on a multiplex but also, if necessary, by photopolygonometry. At present, this method is widely used for the preparation of topographic maps at all scales.

The method of photopolygonometry comprises two stages: determination of plan coordinates of radial centers and determination of the plan position of transformation points. The first step is performed analytically from measured bases and angles between the initial radials, and the second stage is then performed by graphic constructions.



As the radial centers for each aerial photograph, the position of the arbitrary nadir points are taken. These are obtained from eq. (121), after determining the elements of relative orientation and calculating the arbitrary angles of tilt. The position of the arbitrary nadir points is plotted on the photograph with a square-grid tracing sheet of the type shown in Fig. 198; each division of the tracing sheet

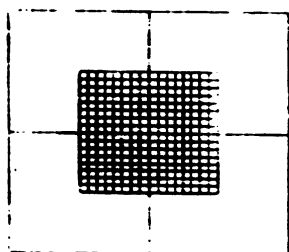


Fig. 198 - Grid Sheet for Construction of the Arbitrary Nadir Points

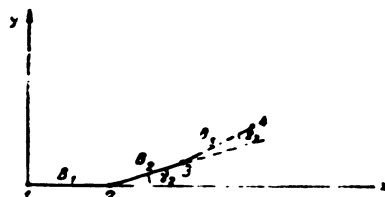


Fig. 199 - Determination of Coordinates of the Radial Centers

corresponds to the arbitrary nadir point for a definite value of the angle of tilt. Therefore, by placing the negative on the tracing sheet and orienting it so that the initial radial coincides with the horizontal lines, and the principal point coincides with the intersection of the base lines of the tracing sheet, the position of the arbitrary nadir point can be pricked on the negative.

The ground coordinates of the radial centers are determined analytically by the equations:

$$\left. \begin{aligned} X_i &= X_{i-1} + \Delta X_{i-1} = X_1 + \sum \Delta X \\ Y_i &= Y_{i-1} + \Delta Y_{i-1} = Y_1 + \sum \Delta Y \end{aligned} \right\} \quad (126)$$

The values of ΔX and ΔY may be obtained from Fig. 199. Let the origin of coord-

ordinates of the traverse coincide with the first radial center, and let the direction of the XX axis be the first initial radial. Then, the ground coordinates of all radial centers will take the following values

$$\begin{aligned}
 X_1 &= 0; & Y_1 &= 0; \\
 X_2 &= \Delta X_1 = B_1; & Y_2 &= 0; \\
 X_3 &= X_2 + \Delta X_2 = B_1 + B_2 \cos \gamma_2; & Y_3 &= Y_2 + \Delta Y_2 = B_2 \sin \gamma_2; \\
 X_4 &= X_3 + \Delta X_3 = B_1 + B_2 \cos \gamma_2 + B_3 \cos (\gamma_2 + \gamma_3); \\
 Y_4 &= Y_3 + \Delta Y_3 = B_2 \sin \gamma_2 + B_3 \sin (\gamma_2 + \gamma_3); \\
 X_n &= X_{n-1} + \Delta X_{n-1} = B_1 + B_2 \cos \gamma_2 + \dots + B_{n-1} \cos (\gamma_2 + \gamma_3 + \dots + \gamma_{n-1}); \\
 Y_n &= Y_{n-1} + \Delta Y_{n-1} = B_2 \sin \gamma_2 + \dots + B_{n-1} \sin (\gamma_2 + \gamma_3 + \dots + \gamma_{n-1})
 \end{aligned}$$

whence

$$\left. \begin{aligned}
 \Delta X_i &= B_i \cos (\gamma_1 + \gamma_2 + \dots + \gamma_i) \\
 \Delta Y_i &= B_i \sin (\gamma_1 + \gamma_2 + \dots + \gamma_i)
 \end{aligned} \right\} \quad (127)$$

Consequently, for calculating the increments of coordinates, we must know the values of the bases B and the angles γ between the initial radials must be known. The values of the bases are determined on the basis of Fig. 200 which shows strictly horizontal photographs for the case that all centers of projection are located in a single horizontal plane. Since the photographs have a 60% end lap, it follows that the contours coinciding with the nadir points of all photographs will be mapped simultaneously on the three photographs. Then, if Z_1 , Z_2 , Z_3 , and Z_4 are the height of the camera station, p the distance on the photograph from the nadir point to the image of the nadir point on the following photograph, while p_r denotes the distance on the photograph from its nadir point to the image of the nadir point of the preceding photograph, we obtain:

$$\left. B_1 = \frac{Z_2}{f_1} p_{r1} = \frac{Z_1}{f_k} p_{r1} \right\}$$

$$\left. \begin{aligned} B_2 &= \frac{Z_3}{f_k} p l_2 = \frac{Z_2}{f_k} p r_2 \\ &\dots\dots\dots \\ B_{n-1} &= \frac{Z_n}{f_k} p l_{n-1} = \frac{Z_{n-1}}{f_k} p r_{n-1} \end{aligned} \right\} \quad (128)$$

In this way, if the height of the camera station Z is known, then, to determine

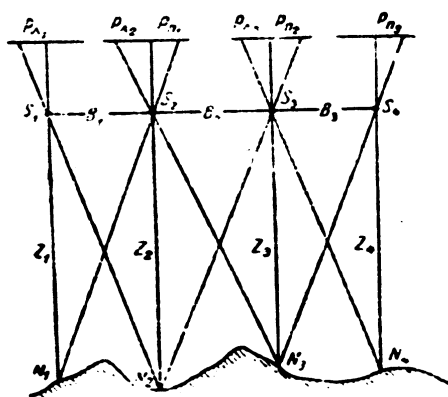


Fig. 200 - Determination of the Photographic Bases

the bases, the distances between the nadir points on the photographs must be known, provided that they strictly correspond to a horizontal position of the photographs and that all the centers of projection lie in a horizontal plane.

Accordingly, we must first pass from the distance p' , measured between the nadir points of the tilted photograph, to the distance p . Since all centers of projection

lie in a plane insignificantly tilted with respect to the horizontal, the segment p can be determined from a straight line parallel to the line S_1S_2 . Then (Fig. 201), the tilt of the optical axis of the camera with respect to the perpendicular to the line S_1S_2 will be τ , if a section of the photographs and the ground is considered a principal base plane containing the radials S_1S_2 and S_1O_1 .

Transition from the distance p' to the distance p in this case will be effected by eq. (10) in which it may be assumed that $v = 90^\circ$. Then,

$$p = p' + \frac{p'^2}{f_k} \sin \tau \quad (129)$$

In expression (129), p' is measured on the photograph, f_k is known, and the

angle τ is obtained by determining the elements of relative orientation, as indicated in Section 103.

113. Construction of a Photopolygonometric Traverse

Before constructing the photopolygonometric traverse, first the elements of relative orientation must be defined (Sect. 103) to obtain the angles τ and to calculate

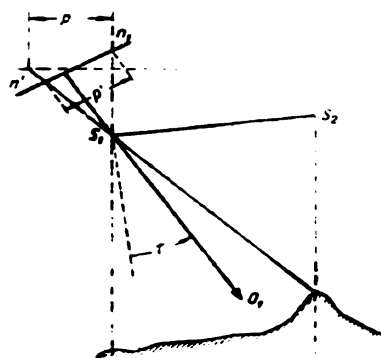


Fig. 201 - Determination of the Correction for Horizontal Parallax

the position of the arbitrary nadir points (see Sect. 109). In addition, the position of the center of rotation of the stereocomparator plate holder must be determined with high accuracy, since the angles between the initial radials are measured instead of being constructed graphically.

Then, the photographs are successively placed on the carriers of the stereocomparator (or the precision stereometer with the correction devices in zero position) and are centered at the arbitrary nadir points. The photographs

are oriented along the initial radials by a method analogous to that described in Section 82. After the photographs have been oriented, the value of χ is read on the scale which fixes the rotation of the carrier in its plane.

The floating mark of the stereocomparator is then brought into contact with the arbitrary nadir point of the right photograph and readings taken on the x scale of the instrument and on the scale of the horizontal parallax screw. The same superposition of the floating mark with the arbitrary nadir point of the left photograph then allows us to take new readings on the x scale and the scale of the horizontal

parallax screw. Then, the measured value of the base is found from the relation

$$\left. \begin{aligned} p'_l &= x_2 - x_1 \\ p'_r &= x_2 - x_1 + \Delta p \end{aligned} \right\} \quad (130)$$

where x_2 is the reading on the x scale when the floating mark is matched with the right arbitrary nadir point; x_1 is the same reading for the left arbitrary nadir point; and Δp is the change in reading on the horizontal parallax screw.

After the measurements have been made on the first stereo pair, the left, or first, photograph is removed from the stereocomparator carrier and the third photograph substituted for it. The second and third photographs are then turned through 180° and oriented in the initial direction. The reading of χ' on the scale, fixing the rotation of the plate holder in its plane, permits calculating the angle between the initial radials from the equation

$$\gamma = \chi' - \chi - 180^\circ \quad (131)$$

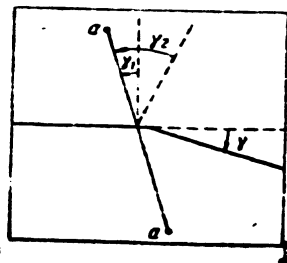
In the same way, the following angles between the initial radials are determined, together with the measured values of the bases.

If the plate holder on the stereocomparator cannot be turned through 360° , then the angles between the initial radials are measured with the aid of auxiliary points. In this case (Fig. 202), any two points (not necessarily contour points) are pricked in a direction roughly perpendicular to the direction of the base line. Then, after orientation of the first pair along the initial radial and after measurement of the base, a reading is taken on the scale of the plate holder, which is then so turned that, when the binocular microscope is displaced along the yy axis, the mark will pass through the two pricked points. A new reading on the scale of the plate holder then gives the angle γ , between the perpendicular to the initial radial and the auxiliary line aa .

For processing the second pair, both photographs are removed from the stereo-

comparator plate holders, the second photograph is substituted for the first photograph, and the third for the second.

The new orientation along the initial radial and the location of the plate holder in its plane, until (on shifting the binocular microscope) the mark passes



through the same pricked points *aa*, makes it possible to measure the angle γ_2 . This angle γ_2 is measured between the perpendicular to the new initial radial and the auxiliary line. In that case, the desired angle γ between the initial radials is given by the following equation:

$$\gamma = \gamma_2 - \gamma_1 \quad (132)$$

Fig. 202 - Measurement of the Angle between Initial Radials

The coordinates of the vertexes of the photopolygonometric traverse are calculated from a record whose standard form is presented in the

Table given below.

In this record, the first entry is the denominator m of the scale ratio of the photopolygonometric traverse, followed by the focal length f_k of the mapping camera and the same focal length f'_k , corrected for the systematic deformation of the film. In Column 2 are entered the measured horizontal parallaxes, in Column 4 the corrections for the influence of the angle τ , and in Column 5 the corrected value of the horizontal parallax. Column 6 will show the previously known flight altitude and Column 9, the measured angle between the initial radials. These data permit calculation of the length of the bases (Column 8), the azimuth of the bases (Column 10), the increments of the coordinates and the x and y coordinates.

Then, a coordinatograph is used for plotting, from their calculated coordinates, the position of all directional vertexes on the plotting board, which is cemented to a rigid base (aluminum or plywood).

114. Determination of the Position of Transformation Points

The plane position of the transformation points is found graphically as a result of the intersection of at least two radials produced from the vertexes of the photopolygonometric traverse. For this purpose, the same points are picked on each photograph that were used in constructing the plan phototriangulation network; these

Record for Calculating the Coordinates of Points of a Photopolygonometric Traverse

$$m = 25,000; f_k = 68.62 \text{ mm}; f'_k = f_k c = 68.58 \text{ mm}$$

Photograph No.	p'_1	τ	$\Delta b = \frac{p'_1}{f'_k} \tau$	$p = p'_1 + \Delta b$	H_r	$k = \frac{H_r}{f'_k}$	$b = bp$	γ	A	$\Delta x = b \cos A$	$\Delta y = b \sin A$	X	Y
1	2	3	4	5	6	7	8	9	10	11	12	13	14
6361												0	0.00
6362	66.41	+37'	+ 0.69	67.10	1726	1.0067	67.65	-48'	0	67.65	0	+67.65	0.00
6363	67.84	-15	- 0.27	67.57	1745	1.0177	68.77	+25'	-48'	68.77	-0.96	+136.42	-0.96

are then copied on tracing paper on which the radials from the arbitrary nadir point are plotted. These wax tracings are placed on the plotting board on which the coordinates of the photopolygonometric traverse had been plotted, in such a way that the radial center of the tracing paper coincides with the given point on the strip, while the initial radials pass through the neighboring points of the traverse. The independent orientation of each tracing permits a check as to the accuracy of determining the coordinates of the traverse vertexes, since there should be no closure errors at the tie points. The intersection of the radial lines at the transformation points makes it possible to mark their position on the board.

Since the transformation points so selected are common to adjacent flight strips, the individual flight strips may be tied-in with the aid of these points, in which case such a connection does not require preliminary reduction, in view of the fact that all the flight strips are constructed at a single specified scale. The

main advantage of photopolygonometry over other methods of photogrammetric densification of the basic horizontal control net is that each base is independently determined from the previously known flight altitude. For this reason, the cumulation of errors in the traverses of a photopolygonometric survey is relatively low, allowing considerable reduction in the amount of geodetic field work.